2
Solving Linear Equations and Inequalities

GOALS OF THIS CHAPTER
The major emphasis of this chapter is to teach you how to solve linear equations. To be successful in solving linear equations, you need to have a thorough understanding of adding, subtracting, multiplying, and dividing real numbers. This material was discussed in Chapter 1.

The first four sections of this chapter will give you the building blocks you will need for solving linear equations. Section 2.5 combines the material previously presented to teach you how to solve a variety of linear equations. In the last few sections of this chapter, you will learn about formulas, ratios, proportions, and solving linear inequalities.

You will be using principles learned in this chapter throughout the book and in real life.

2.1 Combining Like Terms
2.2 The Addition Property of Equality
2.3 The Multiplication Property of Equality
2.4 Solving Linear Equations with a Variable on Only One Side of the Equation
   Mid-Chapter Test: Sections 2.1–2.4
2.5 Solving Linear Equations with the Variable on Both Sides of the Equation
2.6 Formulas
2.7 Ratios and Proportions
2.8 Inequalities in One Variable
   Chapter 2 Summary
   Chapter 2 Review Exercises
   Chapter 2 Practice Test
   Cumulative Review Test

PROPORTIONS ARE VERY USEFUL in everyday living. In Exercise 65 on page 162, we use a proportion to determine how many cups of onions are needed for a recipe.
2.1 Combining Like Terms

Identify terms.

Identify like terms.

Combine like terms.

Use the distributive property.

Remove parentheses when they are preceded by a plus or minus sign.

Simplify an expression.

Identify Terms

In Section 1.3, we indicated that letters called variables are used to represent numbers. A variable can represent a variety of different numbers.

As was indicated in Chapter 1, an expression (sometimes referred to as an algebraic expression) is a collection of numbers, variables, grouping symbols, and operation symbols.

Examples of Expressions

When an algebraic expression consists of several parts, the parts that are added are called the terms of the expression. Consider the expression The expression can be written as and so the expression has three terms: \(2x\), \(x\), and \(3\). The expression also has three terms: \(2xy\), \(xy\), and \(x\).

When listing the terms of an expression, it is not necessary to list the sign at the beginning of a term.

Expression Terms

\[3y, 7, x, 4, 2x^3\]

The numerical part of a term is called its numerical coefficient or simply its coefficient. In the term \(6x\), the 6 is the numerical coefficient. Note that \(6x\) means the variable \(x\) is multiplied by 6.

<table>
<thead>
<tr>
<th>Term</th>
<th>Numerical Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5x)</td>
<td>5</td>
</tr>
<tr>
<td>(-\frac{1}{2}x)</td>
<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>(4(x - 3))</td>
<td>4</td>
</tr>
<tr>
<td>(\frac{2x}{3})</td>
<td>(\frac{2}{3}) since (\frac{2x}{3}) means (\frac{2}{3}x)</td>
</tr>
<tr>
<td>(\frac{x + 4}{3})</td>
<td>(\frac{1}{3}) since (\frac{x + 4}{3}) means (\frac{1}{3}(x + 4))</td>
</tr>
</tbody>
</table>

Whenever a term appears without a numerical coefficient, we assume that the numerical coefficient is 1.

Examples

\(x\) means \(1x\)

\(x^2\) means \(1x^2\)

\(xy\) means \(1xy\)

\((x + 2)\) means \(1(x + 2)\)

\(-x\) means \(-1x\)

\(-x^2\) means \(-1x^2\)

\(-xy\) means \(-1xy\)

\(-(x + 2)\) means \(-1(x + 2)\)
Section 2.1 Combining Like Terms

If an expression has a term that is a number (without a variable), we refer to that number as a constant term, or simply a constant. In the expression \( x^2 + 3x - 4 \), the \(-4\) is a constant term, or a constant.

2 Identify Like Terms

Like terms (also called similar terms) are terms that have the same variables with the same exponents, respectively. Constants, such as 4 and \(-6\), are like terms. Some examples of like terms and unlike terms follow. Note that if two terms are like terms, only their numerical coefficients may differ.

<table>
<thead>
<tr>
<th>Like Terms</th>
<th>Unlike Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3x, \ -4x)</td>
<td>(3x, \ 2)</td>
</tr>
<tr>
<td>(4y, \ 6y)</td>
<td>(3x, \ 4y)</td>
</tr>
<tr>
<td>(5, \ -6)</td>
<td>(x, \ 3)</td>
</tr>
<tr>
<td>(3(x + 1), \ -2(x + 1))</td>
<td>(2x, \ 3xy)</td>
</tr>
<tr>
<td>(3x^2, \ 4x^2)</td>
<td>(3x, \ 4x^2)</td>
</tr>
<tr>
<td>(5ab, \ 2ab)</td>
<td>(4a, \ 2ab)</td>
</tr>
</tbody>
</table>

**EXAMPLE 1** Identify any like terms.

a) \(2x + 3x + 4\)  
   b) \(2x + 3y + 2\)  
   c) \(x + 3 + y - \frac{1}{2}\)  
   d) \(x + 3x^2 - 4x^2\)  
   e) \(5x - x + 6\)  
   f) \(3 - 2x + 4x - 6\)  
   g) \(12 + x^2 - x + 7\)

**Solution**

a) \(2x\) and \(3x\) are like terms.  
   b) There are no like terms.  
   c) \(3\) and \(-\frac{1}{2}\) are like terms.  
   d) \(3x^2\) and \(-4x^2\) are like terms.  
   e) \(3x\) and \(-x\) (or \(-1x\)) are like terms.  
   f) \(3\) and \(-6\) are like terms; \(-2x\) and \(4x\) are like terms.  
   g) \(12\) and \(7\) are like terms.

Now Try Exercise 5

3 Combine Like Terms

We often need to simplify expressions by combining like terms. To combine like terms means to add or subtract the like terms in an expression. To combine like terms, we can use the procedure that follows.

**To Combine Like Terms**

1. Determine which terms are like terms.
2. Add or subtract the coefficients of the like terms.
3. Multiply the number found in step 2 by the common variable(s).

Examples 2 through 7 illustrate this procedure.
EXAMPLE 2  Combine like terms: $5x + 4x$.

Solution  $5x$ and $4x$ are like terms with the common variable $x$. Since $5 + 4 = 9$, then $5x + 4x = 9x$.  

EXAMPLE 3  Combine like terms: $\frac{3}{5}x - \frac{2}{3}x$.

Solution  Since $\frac{3}{5} - \frac{2}{3} = \frac{9}{15} - \frac{10}{15} = -\frac{1}{15}$, then $\frac{3}{5}x - \frac{2}{3}x = -\frac{1}{15}x$.  

EXAMPLE 4  Combine like terms: $6.47b - 8.39b$.

Solution  Since $6.47 - 8.39 = -1.92$, then $6.47b - 8.39b = -1.92b$.  

EXAMPLE 5  Combine like terms: $3x + x + 5$.

Solution  The $3x$ and $x$ are like terms.

$3x + x + 5 = 3x + 1x + 5 = 4x + 5$  

Because of the commutative property of addition, the order of the terms in the answer is not critical. Thus, $5 + 4x$ is also an acceptable answer to Example 5. When writing answers, we generally list the terms containing variables in alphabetical order from left to right, and list the constant term last.

The commutative and associative properties of addition will be used to rearrange the terms in Examples 6 and 7.

EXAMPLE 6  Combine like terms: $3b + 6a - 5 - 2a$.

Solution  The only like terms are $6a$ and $-2a$.

$3b + 6a - 5 - 2a = 6a - 2a + 3b - 5$  

Rearrange terms  

$= 4a + 3b - 5$  

Combine like terms  

EXAMPLE 7  Combine like terms: $-2x^2 + 3y - 4x^2 + 3 - y + 5$.

Solution  

$-2x^2$ and $-4x^2$ are like terms.  

$3y$ and $-y$ are like terms.  

$3$ and $5$ are like terms.  

Grouping the like terms together gives  

$-2x^2 + 3y - 4x^2 + 3 - y + 5 = -2x^2 - 4x^2 + 3y - y + 3 + 5$  

$= -6x^2 + 2y + 8$  

Now Try Exercise 29
Use the Distributive Property

We introduced the distributive property in Section 1.10. Because this property is so important, we will study it again. But before we do, let’s briefly review the subtraction of real numbers. Recall from Section 1.7 that

\[ 6 - 3 = 6 + (-3) \]

In general,

**Subtraction of Real Numbers**

For any real numbers \( a \) and \( b \),

\[ a - b = a + (-b) \]

We will use the fact that \( a + (-b) \) means \( a - b \) in discussing the distributive property.

**Distributive Property**

For any real numbers \( a, b \), and \( c \),

\[ a(b + c) = ab + ac \]

**EXAMPLE 8** Use the distributive property to remove parentheses.

a) \( 2(x + 4) \)

b) \( -5(p + 3) \)

**Solution**

a) \( 2(x + 4) = 2x + 2(4) = 2x + 8 \)

b) \( -5(p + 3) = -5p + (-5)(3) = -5p + (-15) = -5p - 15 \)

Note in part b) that, instead of leaving the answer \(-5p + (-15)\), we wrote it as \(-5p - 15\), which is the proper form of the answer.

**EXAMPLE 9** Use the distributive property to remove parentheses.

a) \( 3(x - 2) \)

b) \( -2(4x - 3) \)

**Solution**

a) By the definition of subtraction, we may write \( x - 2 \) as \( x + (-2) \).

\[ 3(x - 2) = 3[x + (-2)] = 3x + 3(-2) = 3x + (-6) = 3x - 6 \]

b) \( -2(4x - 3) = -2[4x + (-3)] = -2(4x) + (-2)(-3) = -8x + 6 \)

The distributive property is used often in algebra, so you need to understand it well. You should understand it so well that you will be able to simplify an expression using the distributive property without having to write down all the steps that we listed in working Examples 8 and 9. Study closely the Helpful Hint that follows.
Helpful Hint

With a little practice, you will be able to eliminate some of the intermediate steps when you use the distributive property. When using the distributive property, there are eight possibilities with regard to signs. Study and understand the eight possibilities that follow.

### Examples of the Expanded Distributive Property

<table>
<thead>
<tr>
<th>Positive Coefficient</th>
<th>Negative Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (2(x) = 2x)</td>
<td>e) ((-2)(x) = -2x)</td>
</tr>
<tr>
<td>(2(x + 3) = 2x + 6)</td>
<td>((-2)(x + 3) = -2x - 6)</td>
</tr>
<tr>
<td>(2 + 3 = +6)</td>
<td>((-2) + 3 = -6)</td>
</tr>
<tr>
<td>b) (2(x) = 2x)</td>
<td>f) ((-2)(x) = -2x)</td>
</tr>
<tr>
<td>(2(x - 3) = 2x - 6)</td>
<td>((-2)(x - 3) = -2x + 6)</td>
</tr>
<tr>
<td>(2(-3) = -6)</td>
<td>((-2)(-3) = +6)</td>
</tr>
<tr>
<td>c) (2(-x) = -2x)</td>
<td>g) ((-2)(-x) = 2x)</td>
</tr>
<tr>
<td>(2(-x + 3) = -2x + 6)</td>
<td>((-2)(-x + 3) = 2x - 6)</td>
</tr>
<tr>
<td>(2(+3) = +6)</td>
<td>((-2)(+3) = -6)</td>
</tr>
<tr>
<td>d) (2(-x) = -2x)</td>
<td>h) ((-2)(-x) = 2x)</td>
</tr>
<tr>
<td>(2(-x - 3) = -2x - 6)</td>
<td>((-2)(-x - 3) = 2x + 6)</td>
</tr>
<tr>
<td>(2(-3) = -6)</td>
<td>((-2)(-3) = +6)</td>
</tr>
</tbody>
</table>

The distributive property can be expanded as follows:

\[ a(b + c + d + \cdots + n) = ab + ac + ad + \cdots + an \]

**Examples of the Expanded Distributive Property**

\[ 3(x + y + z) = 3x + 3y + 3z \]

\[ 2(x + y - 3) = 2x + 2y - 6 \]

**EXAMPLE 10** Use the distributive property to remove parentheses.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (4(x - 3))</td>
<td>b) (-6(5y - 1))</td>
</tr>
<tr>
<td>c) (-\frac{1}{2}(4r + 5))</td>
<td>d) (-7(2x + 4y - 9z))</td>
</tr>
</tbody>
</table>

**Solution**

- a) \(4(x - 3) = 4x - 12\)
- b) \(-6(5y - 1) = -30y + 6\)
- c) \(-\frac{1}{2}(4r + 5) = -2r - \frac{5}{2}\)
- d) \(-7(2x + 4y - 9z) = -14x - 28y + 63z\)

**EXAMPLE 11** Use the distributive property to remove parentheses from the expression \((2x - 8y)4\).

**Solution** We distribute the 4 on the right side of the parentheses over the terms within the parentheses.

\[(2x - 8y)4 = 2x(4) - 8y(4) = 8x - 32y\]

Example 11 could have been rewritten as \(4(2x - 8y)\) by the commutative property of multiplication, and then the 4 could have been distributed from the left to obtain the same answer, \(8x - 32y\).
Helpful Hint

Students sometimes try to use the distributive property when it cannot be used. For the distributive property to be used, there must be a + or − between the terms within parentheses and the terms within parentheses must be multiplied by some number or expression. Study the following correct simplifications carefully.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(2xy) = 8xy</td>
<td>(distributive property is not used)</td>
</tr>
<tr>
<td>4(2x + y) = 8x + 4y</td>
<td>(distributive property is used)</td>
</tr>
<tr>
<td>(2x + y) - 4 = 2x + y - 4</td>
<td>(distributive property is not used)</td>
</tr>
<tr>
<td>(2x + y)(-4) = -8x - 4y</td>
<td>(distributive property is used)</td>
</tr>
</tbody>
</table>

5 Remove Parentheses When They Are Preceded by a Plus or Minus Sign

In the expression (4x + 3), how do we remove parentheses? Recall that the coefficient of a term is assumed to be 1 if none is shown. Therefore, we may write

\[
(4x + 3) = 1(4x + 3) \\
= 1(4x) + (1)(3) \\
= 4x + 3
\]

Note that (4x + 3) = 4x + 3. When no sign or a plus sign precedes parentheses, the parentheses may be removed without having to change the expression inside the parentheses.

Examples

\[
(x + 3) = x + 3 \\
(2x - 3) = 2x - 3 \\
+(2x - 5) = 2x - 5 \\
+(x + 2y - 6) = x + 2y - 6
\]

Now consider the expression -(4x + 3). How do we remove parentheses in this expression? Here, the coefficient in front of the parentheses is -1, so each term within the parentheses is multiplied by -1.

\[
-(4x + 3) = -1(4x + 3) \\
= -1(4x) + (-1)(3) \\
= -4x + (-3) \\
= -4x - 3
\]

Thus, -(4x + 3) = -4x - 3. When a minus sign precedes parentheses, the signs of all the terms within the parentheses are changed when the parentheses are removed.

Examples

\[
-(x + 4) = -x - 4 \\
-(-2x + 3) = 2x - 3 \\
-(5x - y + 3) = -5x + y - 3 \\
-(-4c - 3d - 5) = 4c + 3d + 5
\]

6 Simplify an Expression

Combining what we learned in the preceding discussions, we have the following procedure for simplifying an expression.

To Simplify an Expression

1. Use the distributive property to remove any parentheses.
2. Combine like terms.
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**EXAMPLE 12** Simplify $6 - (2x + 3)$.

**Solution**

$6 - (2x + 3) = 6 - 2x - 3$  Use the distributive property.

$= -2x + 3$  Combine like terms.

**Note:** $3 - 2x$ is the same as $-2x + 3$; however, we generally write the term containing the variable first.  

Now Try Exercise 89

**EXAMPLE 13** Simplify $-\left(\frac{2}{3}x - \frac{1}{4}\right) + 3x$.

**Solution**

$-\left(\frac{2}{3}x - \frac{1}{4}\right) + 3x = -\frac{2}{3}x + \frac{1}{4} + 3x$  Distributive property

$= -\frac{2}{3}x + 3x + \frac{1}{4}$  Rearrange terms.

$= -\frac{2}{3}x + \frac{9}{3}x + \frac{1}{4}$  Write $x$ terms with the LCD, 3.

$= \frac{7}{3}x + \frac{1}{4}$  Combine like terms.

Now Try Exercise 101

Notice in Example 13 that $\frac{7}{3}x$ and $\frac{1}{4}$ could not be combined because they are not like terms.

**EXAMPLE 14** Simplify $\frac{3}{4}x^2 + \frac{1}{3}(5x - 2)$.

**Solution**

$\frac{3}{4}x^2 + \frac{1}{3}(5x - 2) = \frac{3}{4}x^2 + \frac{1}{3}(5x) + \frac{1}{3}(-2)$  Distributive property

$= \frac{3}{4}x^2 + \frac{5}{3}x - \frac{2}{3}$

$= \frac{9}{12}x^2 + \frac{20}{12}x - \frac{2}{3}$  Write $x$ terms with the LCD, 12.

$= \frac{29}{12}x^2 - \frac{2}{3}$  Combine like terms.

Now Try Exercise 103

**EXAMPLE 15** Simplify $3(2a - 5) - 3(b - 6) - 4a$.

**Solution**

$3(2a - 5) - 3(b - 6) - 4a = 6a - 15 - 3b + 18 - 4a$  Distributive property

$= 6a - 4a - 3b - 15 + 18$  Rearrange terms.

$= 2a - 3b + 3$  Combine like terms.

Now Try Exercise 107

**Helpful Hint**

Keep in mind the difference between the concepts of term and factor. When two or more expressions are multiplied, each expression is a factor of the product. For example, since $4 \cdot 3 = 12$, the 4 and the 3 are factors of 12. Since $3 \cdot x = 3x$, the 3 and the $x$ are factors of $3x$. Similarly, in the expression $5xyz$, the 5, $x$, $y$, and $z$ are all factors.

In an expression, the parts that are added are the terms of the expression. For example, the expression $2x^2 + 3x - 4$, has three terms, $2x^2$, $3x$, and $-4$. Note that the terms of an expression may have factors. For example, in the term $2x^2$, the 2 and the $x^2$ are factors because they are multiplied.
EXERCISE SET 2.1

Concept/Writing Exercises

1. a) What are the terms of an expression?
   b) What are the terms of $3x - 4y - 5$?
   c) What are the terms of $6xy + 3x - y - 9$?

2. a) What is the name given to the numerical part of a term?
   List the coefficient of the following terms.
   b) $4x$
   c) $x$
   d) $-x$
   e) $\frac{3x}{5}$
   f) $\frac{4}{7}(3t - 5)$

3. Consider the expression $2x - 5$.
   a) What is the $x$ called?
   b) What is the $-5$ called?
   c) What is the 2 called?

4. a) What are like terms? Determine whether the following are like terms.
    If not, explain why.
    b) $3x, 4y$
    c) $7, -2$
    d) $5x^2, 2x$
    e) $4x, -5xy$

5. Determine whether the following are like terms.
   If not, explain why.
   a) $5x, -7x$
   b) $7y, 2$
   c) $-3(t + 2), 6(t + 2)$
   d) $4pq, -9pq$

6. a) When no sign or a plus sign precedes an expression within parentheses, explain how to remove parentheses.
    b) Write $+ (x - 8)$ without parentheses.

7. a) When a minus sign precedes an expression within parentheses, explain how to remove parentheses.
    b) Write $- (x - 8)$ without parentheses.

8. a) What are the factors of an expression?
    b) Explain why $3$ and $x$ are factors of $3x$.
    c) Explain why $5, x,$ and $y$ are all factors of the expression $5xy$.

Practice the Skills

Combine like terms when possible. If not possible, rewrite the expression as is.

9. $6x + 3x$
10. $4x - 5x$
11. $3x + 6$
12. $4x + 3y$
13. $y + 3 + 4y$
14. $4x - 7x + 4$
15. $\frac{3}{4}a - \frac{6}{11}a$
16. $\frac{3}{4}p - \frac{2}{7}p$
17. $2 - 6x + 5$
18. $-7 - 4m - 6$
19. $-2w - 3w + 5$
20. $-8y - 4y - 7$
21. $-x + 2 - x - 2$
22. $8x - 2y - 1 - 3x$
23. $3 + 6x - 3 - 6x$
24. $y - 2y + 5$
25. $5 + 2x - 4x + 6$
26. $5x - 3s - 2s$
27. $4r - 6 - 6r - 2$
28. $-6r + 5 + 2t - 9$
29. $3x^2 - 9y^2 + 7x^2 - 5 - y^2 - 2$
30. $-4x^2 - 6y - 3x^2 + 6 - y - 1$
31. $-2x + 4x - 3$
32. $4 - x + 4x - 8$
33. $b + 4 + \frac{3}{5}$
34. $\frac{3}{5} x + 2 + x$
35. $5.1n + 6.42 - 4.3n$
36. $2x^3 + 3y^2 + 4x + 5y^2$
37. $\frac{1}{2} a + 3b + 1$
38. $x + \frac{1}{2} y - \frac{3}{8} y$
39. $13.4x + 1.2x + 8.3$
40. $-4x^2 - 3.1 - 5.2$
41. $-x^2 + 2x^2 + y$
42. $1 + x^2 + 6 - 3x^2$
43. $2x - 7y - 5x + 2y$
44. $3x - 7 - 9 + 4x$
45. $4 - 3n^2 + 9 - 2n$
46. $9x + y - 2 - 4x$
47. $-19.36 + 40.02x + 12.25 - 18.3x$
48. $52x - 52x - 63.5 - 63.5$
49. $\frac{3}{5} x - 3 - \frac{7}{4} x - 2$
50. $\frac{1}{7} y - 4 + \frac{3}{4} x - \frac{1}{5} y$
51. $5w^2 + 2w^2 + w + 3$
52. $4p^2 - 3p^2 + 2p - 5p$
53. $2x - 5z^2 - 2z^3 - z^2$
54. $5ab - 3ab$
55. $6z^2 - 6xy + 3y^2$
56. $x^2 - 3xy - 2xy + 6$
57. $4a^2 - 5ab + 6ab + b^2$
58. $4b^2 - 8bc + 5bc + c^2$

Use the distributive property to remove parentheses.

59. $5(x + 2)$
60. $2(-y + 5)$
61. $5(x + 4)$
62. $-2(y + 8)$
63. $3(x - 6)$
64. $-2(x - 4)$
65. $-\frac{1}{2}(2x - 4)$
66. $-4(x + 6)$
67. $1(-4 + x)$

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68. \( \frac{2}{3}(m - 9) \)  
69. \( \frac{4}{5}(x - 5) \)  
70. \( 5(x - y + 5) \)

71. \(-0.3(3x + 5)\)  
72. \(- (x - 3)\)  
73. \(-\frac{1}{3}(3r - 12)\)

74. \(-2(x + y - z)\)  
75. \(0.7(2x + 0.5)\)  
76. \(- (x + 4y)\)

77. \(- (-x + y)\)  
78. \((3x + 4y - 6)\)  
79. \(- (2x + 4y - 8)\)

80. \(-3(2a + 3b - 7)\)  
81. \(1.1(3.1x - 5.2y + 2.8)\)  
82. \(-4(-2m - 3n + 8)\)

83. \((2x - 9y)5\)  
84. \((8b - 1)7\)  
85. \((x + 3y - 9)\)

86. \((-p + 2q - 3)\)  
87. \(-3(-x + 2y + 4)\)  
88. \(2.3(1.6x + 5.1y - 4.1)\)

Simplify.

90. \(7 - (2x - 9)\)  
93. \(6x + 2(4x + 9)\)

91. \(-2(3 - x) + 7\)  
94. \(3(x + y) + 2y\)

92. \(- (3x - 3) + 5\)  
96. \(6 + (x - 5) + 3x\)

97. \(4(2e - 3) - 3(c - 4)\)  
100. \(- (x - 5) - 3x + 4\)

95. \(2(x - y) + 2x + 3\)  
98. \((2y + 2) + y\)

103. \(\frac{2}{3}x + \frac{1}{2}(5x - 4)\)

104. \(\frac{4}{5}x + \frac{1}{2}(3x - 1)\)  
105. \(- (3x + 4) - (x + 2)\)

106. \(6 - 2(x + 3) + 5x\)

107. \(4(x - 1) + 2(3 - x) - 4\)  
108. \(4(3b - 2) - 5(c - 4) - 6b\)

109. \(4(m + 3) - 4m - 12\)

110. \(-3(a + 2b) + 3(a + 2b)\)  
111. \(0.4 - (x + 5) + 0.6 - 2\)

112. \(4 - (2 - x) + 3x\)

113. \(4 + (3x - 4) - 5\)  
114. \(2y - 6(y - 2) + 3\)

115. \(4(x + 2) - 3(x - 4) - 5\)

116. \(6 - (a - 5) - (2b + 1)\)  
117. \(-0.2(6 - x) - 4(1.4 + 0.4)\)

118. \(-5(2y - 8) - 3(1 + x) - 7\)

119. \(-6x + 7y - (3 + x) + (x + 3)\)  
120. \(3(t - 2) - 2(t + 4) - 6\)

121. \(\frac{1}{2}(x + 3) + \frac{1}{3}(3x + 6)\)

Problem Solving

If \( \square + \square + \triangle + \square + \bigcirc\) can be represented as \(3\square + 2\bigcirc\), write an expression to represent each of the following.

123. \(\square + \bigcirc + \triangle + \square + \bigcirc\)

124. \(\bigcirc + \bigcirc + \triangle + \bigcirc + \triangle + \bigcirc\)

125. \(x + y + \triangle + \triangle + x + y + y\)

126. \(2 + x + 2 + \bigcirc + \bigcirc + 2 + y\)

Combine like terms.

127. \(3\triangle + 5\bigcirc - \triangle - 3\bigcirc\)

128. \(8\bigcirc - 4\bigcirc - 2\bigcirc - 3\bigcirc\)

In Exercises 129 and 130, consider the following. The positive factors of 6 are 1, 2, 3, and 6 since

\[
\begin{align*}
1 \cdot 6 &= 6 \\
2 \cdot 3 &= 6
\end{align*}
\]

129. List all the positive factors of 18.

130. List all the positive factors of 24.

Challenge Problems

Simplify.

131. \(4x^2 + 5y^2 + 6(3x^2 - 5y^2) - 4x + 3\)

132. \(2x^2 - 4x + 8x^2 - 3(x + 2) - x^2 - 2\)

133. \(2[3 + 4(x - 5)] - [2 - (x - 3)]\)

134. \(\frac{1}{4}[3 - 2(y + 1)] - \frac{1}{3}[2 - (y - 6)]\)
Cumulative Review Exercises

[1.5] Evaluate.
135. \[ | -7 | \]
136. \[ | -16 | \]

[1.7] Evaluate \[ -4 - 3 - (-6) \].

[1.9] Write a paragraph explaining the order of operations.
138. Evaluate \[ -x^2 + 5x - 6 \] when \( x = -1 \).

2.2 The Addition Property of Equality

1 Identify Linear Equations
A statement that shows two algebraic expressions are equal is called an equation. For example, \( 4x + 3 = 2x - 4 \) is an equation. In this chapter, we learn to solve linear equations in one variable.

Linear Equation
A linear equation in one variable is an equation that can be written in the form

\[ ax + b = c \]

where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \).

Examples of Linear Equations
\[ x + 4 = 7 \]
\[ 2x - 4 = 6 \]

2 Check Solutions to Equations
The solution to an equation is the number or numbers that when substituted for the variable or variables make the equation a true statement. For example, the solution to \( x + 4 = 7 \) is 3. We will shortly learn how to find the solution to an equation, or to solve an equation. But before we do this we will learn how to check the solution to an equation.

The solution to an equation may be checked by substituting the value that is believed to be the solution for the variable in the original equation. If the substitution results in a true statement, your solution is correct. If the substitution results in a false statement, then either your solution or your check is incorrect, and you need to go back and find your error. Try to check all your solutions. Checking the solutions will improve your arithmetic and algebra skills.

When we show the check of a solution we shall use the notation, \( \frac{3}{7} \). This notation is used when we are questioning whether a statement is true. For example, if we use

\[ 2 + 3 \overset{?}{=} 2(3) - 1 \]

we are asking “Does \( 2 + 3 = 2(3) - 1 \)?”

To check whether 3 is the solution to \( x + 4 = 7 \), we substitute 3 for each \( x \) in the equation.

Check:
\[ x = 3 \]
\[ x + 4 = 7 \]
\[ 3 + 4 \overset{?}{=} 7 \]
\[ 7 = 7 \quad \text{True} \]

Since the check results in a true statement, 3 is a solution.
EXAMPLE 1  Consider the equation $2x - 4 = 6$. Determine whether 3 is a solution.

**Solution**  To determine whether 3 is a solution to the equation, we substitute 3 for $x$.

Check:

$\begin{align*}
2x - 4 &= 6 \\
2(3) - 4 &= 6 \\
6 - 4 &= 6 \\
2 &= 6 \\
&= False
\end{align*}$

Since we obtained a false statement, 3 is not a solution.

Now Try Exercise 13

Now check to see if 5 is a solution to the equation in Example 1. Your check should show that 5 is a solution.

We can use the same procedures to check more complex equations, as shown in Examples 2 and 3.

EXAMPLE 2  Determine whether 18 is a solution to the following equation.

$3x - 2(x + 3) = 12$

**Solution**  To determine whether 18 is a solution, we substitute 18 for each $x$ in the equation. If the substitution results in a true statement, then 18 is a solution.

Check:

$\begin{align*}
3x - 2(x + 3) &= 12 \\
3(18) - 2(18 + 3) &= 12 \\
3(18) - 2(21) &= 12 \\
54 - 42 &= 12 \\
12 &= 12 \\
&= True
\end{align*}$

Since we obtained a true statement, 18 is a solution.

Now Try Exercise 17

EXAMPLE 3  Determine whether $-\frac{3}{2}$ is a solution to the following equation.

$3(n + 3) = 6 + n$

**Solution**  In this equation $n$ is the variable. Substitute $-\frac{3}{2}$ for each $n$ in the equation.

Check:

$\begin{align*}
3(n + 3) &= 6 + n \\
3\left(-\frac{3}{2} + 3\right) &= 6 + \left(-\frac{3}{2}\right) \\
3\left(-\frac{3}{2} + \frac{6}{2}\right) &= 12 - \frac{3}{2} \\
3\left(-\frac{3}{2} + \frac{6}{2}\right) &= \frac{9}{2} \\
\frac{9}{2} &= \frac{9}{2} \\
&= True
\end{align*}$

Thus, $-\frac{3}{2}$ is a solution.

Now Try Exercise 23
### Using Your Calculator: Checking Solutions

Calculators can be used to check solutions to equations. For example, to check whether \( \frac{-10}{3} \) is a solution to the equation \( 2x + 3 = 5(x + 3) - 2 \), we perform the following steps.

1. Substitute \( \frac{-10}{3} \) for each \( x \) as shown below.

\[
2x + 3 = 5(x + 3) - 2
\]

\[
2 \left( \frac{-10}{3} \right) + 3 \neq 5 \left( \frac{-10}{3} + 3 \right) - 2
\]

2. Evaluate each side of the equation separately using your calculator. If you obtain the same value on both sides, your solution checks.

#### Scientific Calculator

To evaluate the left side of the equation, press the following keys:

\[
2 \times \left( \left( \frac{10}{3} \right) + 3 \right) \pm \frac{1}{2} = -3.6666667
\]

To evaluate the right side of the equation, \( 5 \left( \frac{-10}{3} + 3 \right) - 2 \), press the following keys:

\[
5 \times \left( \left( \frac{10}{3} \right) + 3 + 3 \right) \pm 2 = -3.6666667
\]

Since both sides give the same value, the solution checks. Note that because calculators differ in their electronics, sometimes the last digit of a calculation will differ.

#### Graphing Calculator

- Left side of equation: \( 2 \left( \left( - \frac{10}{3} \right) + 3 \right) \pm \frac{1}{2} \) ENTER \(-3.6666667\)
- Right side of the equation: \( 5 \left( \left( - \frac{10}{3} \right) + 3 + 3 \right) \pm 2 \) ENTER \(-3.6666667\)

Since both sides give the same value, the solution checks.

### Identify Equivalent Equations

Now that we know how to check a solution to an equation we will discuss solving equations. Complete procedures for solving equations will be given shortly. For now, you need to understand that to solve an equation, it is necessary to get the variable alone on one side of the equal sign. That is, we want to get an equation of the form \( x = \) some number (or \( 1x = \) some number). When we get an equation in this form, we say that we isolate the variable. To isolate the variable, we make use of two properties: the addition and multiplication properties of equality. Look first at Figure 2.1.

Think of an equation as a balanced statement whose left side is balanced by its right side. When solving an equation, we must make sure that the equation remains balanced at all times. That is, both sides must always remain equal. **We ensure that an equation always remains equal by doing the same thing to both sides of the equation.** For example, if we add a number to the left side of the equation, we must add exactly the same number to the right side. If we multiply the right side of the equation by some number, we must multiply the left side by the same number.

When we add the same number to both sides of an equation or multiply both sides of an equation by the same nonzero number, we do not change the solution to the equation, just the form of the equation. Two or more equations with the same solution are called **equivalent equations.** The equations \( 2x - 4 = 2, 2x = 6, \) and \( x = 3 \) are equivalent, since the solution to each is \( 3. \)
Chapter 2 Solving Linear Equations and Inequalities

Check: \( x = 3 \)

\[
\begin{align*}
2x - 4 &= 2 \quad &2x &= 6 \quad &x &= 3 \\
2(3) - 4 &= 2 \quad &2(3) &= 6 \quad &3 &= 3 \quad \text{True} \\
6 - 4 &= 2 \quad &6 &= 6 \quad &\text{True} \\
2 &= 2 &\text{True}
\end{align*}
\]

When solving an equation, we use the addition and multiplication properties to express a given equation as simpler equivalent equations until we obtain the solution.

4 Use the Addition Property to Solve Equations

Now we are ready to define the addition property of equality.

**Addition Property of Equality**

If \( a = b \), then \( a + c = b + c \) for any real numbers \( a, b, \) and \( c \).

This property means that the same number can be added to both sides of an equation without changing the solution. The addition property is used to solve equations of the form \( x + a = b \). To isolate the variable \( x \) in equations of this form, add the opposite or additive inverse of \( a, -a \), to both sides of the equation.

To isolate the variable when solving equations of the form \( x + a = b \), we use the addition property to eliminate the number on the same side of the equal sign as the variable. Study the following examples carefully.

**To Solve, Use the Addition Property to Eliminate the Number**

<table>
<thead>
<tr>
<th>Equation</th>
<th>To Solve, Use the Addition Property to Eliminate the Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x - 4 = -3 )</td>
<td>(-4)</td>
</tr>
<tr>
<td>( x + 5 = 9 )</td>
<td>(5)</td>
</tr>
<tr>
<td>(-3 = k + 7)</td>
<td>(7)</td>
</tr>
<tr>
<td>(-5 = x - 4)</td>
<td>(-4)</td>
</tr>
<tr>
<td>(-6.25 = y + 12.78)</td>
<td>(12.78)</td>
</tr>
</tbody>
</table>

Now let’s work some examples.

EXAMPLE 4 Solve the equation \( x - 4 = -3 \).

**Solution**: To isolate the variable, \( x \), we must eliminate the \(-4\) from the left side of the equation. To do this we add \(4\), the opposite of \(-4\), to both sides of the equation.

\[
\begin{align*}
x - 4 &= -3 \\
x - 4 + 4 &= -3 + 4 & \text{Add 4 to both sides} \\
x &= 1
\end{align*}
\]

Note how the process helps to isolate \( x \).

Check: \( x - 4 = -3 \)

\[
\begin{align*}
1 - 4 &= \not{-3} \\
-3 &= \not{-3} & \text{True}
\end{align*}
\]

\( \text{Now Try Exercise 29} \)

In Example 5, we will not show the check. Space limitations prevent us from showing all checks. However, you should check all of your answers.
EXAMPLE 5  Solve the equation $x + 5 = 9$.

Solution  To solve this equation, we must isolate the variable, $x$. Therefore, we must eliminate the 5 from the left side of the equation. To do this, we add $-5$, the opposite of 5, to both sides of the equation.

\[
x + 5 = 9
\]
\[
x + 5 + (-5) = 9 + (-5)
\]
\[
x + 0 = 4
\]
\[
x = 4
\]

Now Try Exercise 31

In Example 5, we added $-5$ to both sides of the equation. From Section 1.7 we know that $5 + (-5) = 5 - 5$. Thus, we can see that adding a negative 5 to both sides of the equation is equivalent to subtracting a 5 from both sides of the equation. According to the addition property, the same number may be added to both sides of an equation. Since subtraction is defined in terms of addition, the addition property also allows us to subtract the same number from both sides of the equation. Thus, Example 5 could have also been worked as follows:

\[
x + 5 = 9
\]
\[
x + 5 - 5 = 9 - 5
\]
\[
x + 0 = 4
\]
\[
x = 4
\]

Now Try Exercise 31

EXAMPLE 6  Solve the equation $-3 = k + 7$.

Solution  We must isolate the variable, $k$, which is on the right side of the equal sign.

\[
-3 = k + 7
\]
\[
-3 - 7 = k + 7 - 7
\]
\[
-10 = k + 0
\]
\[
-10 = k
\]

Check:

\[
-3 = k + 7
\]
\[
-3 - 10 + 7
\]
\[
-3 = -3
\]

Now Try Exercise 27

Helpful Hint

Remember that our goal in solving an equation is to get the variable alone on one side of the equation. To do this, we add or subtract the number on the same side of the equation as the variable to or from both sides of the equation.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Must Eliminate</th>
<th>Number to Add (or Subtract) to (or from) Both Sides of the Equation</th>
<th>Correct Results</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x - 5 = 8$</td>
<td>$-5$</td>
<td>add 5</td>
<td>$x - 5 + 5 = 8 + 5$</td>
<td>$x = 13$</td>
</tr>
<tr>
<td>$x - 3 = -12$</td>
<td>$-3$</td>
<td>add 3</td>
<td>$x - 3 + 3 = -12 + 3$</td>
<td>$x = -9$</td>
</tr>
<tr>
<td>$2 = x - 7$</td>
<td>$-7$</td>
<td>add 7</td>
<td>$2 + 7 = x - 7 + 7$</td>
<td>$9 = x$ or $x = 9$</td>
</tr>
<tr>
<td>$x + 12 = -5$</td>
<td>$+12$</td>
<td>subtract 12</td>
<td>$x + 12 - 12 = -5 - 12$</td>
<td>$x = -17$</td>
</tr>
<tr>
<td>$6 = x + 4$</td>
<td>$+4$</td>
<td>subtract 4</td>
<td>$6 - 4 = x + 4 - 4$</td>
<td>$2 = x$ or $x = 2$</td>
</tr>
<tr>
<td>$13 = x + 9$</td>
<td>$+9$</td>
<td>subtract 9</td>
<td>$13 - 9 = x + 9 - 9$</td>
<td>$4 = x$ or $x = 4$</td>
</tr>
</tbody>
</table>

Notice that under the Correct Results column, when the equation is simplified by combining terms, the $x$ will become isolated because the sum of a number and its opposite is 0, and $x + 0$ equals $x$. 
EXAMPLE 7  Solve the equation \( -5 = x - 4 \).

Solution  The variable, \( x \), is on the right side of the equation. To isolate the \( x \), we must eliminate the \(-4\) from the right side of the equation. This can be accomplished by adding 4 to both sides of the equation.

\[
\begin{align*}
-5 &= x - 4 \\
-5 + 4 &= x - 4 + 4 & \text{Add 4 to both sides.} \\
-1 &= x + 0 \\
-1 &= x
\end{align*}
\]

Thus, the solution is \(-1\).

Now Try Exercise 37

EXAMPLE 8  Solve the equation \(-6.25 = y + 12.78\).

Solution  The variable, \( y \), is on the right side of the equation. Subtract 12.78 from both sides of the equation to isolate the variable.

\[
\begin{align*}
-6.25 &= y + 12.78 \\
-6.25 - 12.78 &= y + 12.78 - 12.78 & \text{Subtract 12.78 from both sides.} \\
-19.03 &= y + 0 \\
-19.03 &= y
\end{align*}
\]

Thus, the solution is \(-19.03\).

Now Try Exercise 67

Avoiding Common Errors

When solving an equation, our goal is to isolate the variable on one side of the equal sign. Consider the equation \( x + 3 = -4 \). How do we solve it?

\[
\begin{align*}
\text{CORRECT} & \quad \text{INCORRECT} \\
\text{Remove the 3 from the left side of the equation.} & \quad \text{Remove the } -4 \text{ from the right side of the equation.} \\
x + 3 &= -4 & x + 3 &= -4 \\
x + 3 - 3 &= -4 - 3 & x + 3 + 4 &= -4 + 4 \\
x &= -7 & x + 7 &= 0
\end{align*}
\]

Remember, use the addition property to remove the number that is on the same side of the equal sign as the variable.

5 Solve Equations by Doing Some Steps Mentally

Consider the following two problems.

\[
\begin{align*}
a) \quad x - 5 &= 12 & b) \quad 15 &= x + 3 \\
15 &= x + 3 - 3 & 15 - 3 &= x + 3 - 3 \\
x - 5 + 5 &= 12 + 5 & x &= 17 \\
x + 0 &= 12 + 5 & 15 - 3 &= x + 0 \\
x &= 12 & 12 &= x
\end{align*}
\]

Note how the number on the same side of the equal sign as the variable is transferred to the opposite side of the equal sign when the addition property is used. Also note that the sign of the number changes when transferred from one side of the equal sign to the other.
When you feel comfortable using the addition property of equality, you may wish to do some of the steps mentally to reduce some of the written work. For example, the preceding two problems may be shortened as follows:

Shortened Form

\[ a) \quad x - 5 = 12 \]
\[ x - 5 + 5 = 12 + 5 \]
\[ x = 17 \]

Do this step mentally.

\[ b) \quad 15 = x + 3 \]
\[ 15 - 3 = x + 3 - 3 \]
\[ 12 = x \]

Do this step mentally.

EXERCISE SET 2.2

Concept/Writing Exercises

1. What is an equation?
2. a) What is meant by the “solution to an equation”?  
   b) What does it mean to “solve an equation”?
3. Explain how the solution to an equation may be checked.
4. Explain the addition property of equality.
5. What are equivalent equations?
6. To solve an equation we “isolate the variable.”
   a) Explain what this means.
   b) Explain how to isolate the variable in the equations discussed in this section.
7. When solving the equation \( x + 2 = 6 \), would you subtract 6 from both sides of the equation or subtract 2 from both sides of the equation? Explain.
8. When solving the equation \( x - 4 = 6 \), would you add 4 to both sides of the equation or subtract 6 from both sides of the equation? Explain.
9. Give an example of a linear equation in one variable.
10. Explain why the addition property allows us to subtract the same quantity from both sides of an equation.
11. Explain why the following three equations are equivalent.
12. To solve the equation \( x - □ = △ \) for \( x \), do we add □ to both sides of the equation or do we subtract △ from both sides of the equation? Explain.

Practice the Skills

13. Is \( x = 2 \) a solution of \( 4x - 3 = 5? \)
14. Is \( x = -6 \) a solution of \( 2x + 1 = x - 5? \)
15. Is \( x = -3 \) a solution of \( 2x - 5 = 5(x + 2)? \)
16. Is \( x = 1 \) a solution of \( 2(x - 3) = -3(x + 1)? \)
17. Is \( p = -15 \) a solution of \( 2p - 5(p + 7) = 10? \)
18. Is \( k = -2 \) a solution of \( 5k - 6(k - 1) = 8? \)
19. Is \( x = 3.4 \) a solution of \( 3(x + 2) - 3(x - 1) = 9? \)
20. Is \( x = \frac{3}{4} \) a solution of \( x + 5 = 5x + 2? \)
21. Is \( x = \frac{1}{2} \) a solution of \( 4x - 4 = 2x - 2? \)
22. Is \( x = \frac{1}{3} \) a solution of \( 7x + 3 = 2x + 5? \)
23. Is \( x = \frac{11}{2} \) a solution of \( 3(x + 2) = 5(x - 1)? \)
24. Is \( h = 3 \) a solution of \( -(h - 5) = (h - 6) = 3h - 4? \)

Solve each equation and check your solution.

25. \( x + 2 - 7 \)
26. \( x - 4 = 13 \)
27. \( -6 = x + 1 \)
28. \( -5 = x + 4 \)
29. \( x - 4 = -8 \)
30. \( x + 9 = 52 \)
31. \( x + 8 = 17 \)
32. \( x + 16 = -12 \)
33. \( -6 + w = 9 \)
34. \( 3 = 7 + t \)
35. \( 27 = x + 16 \)
36. \( 50 = x - 25 \)
37. \( -18 = x - 14 \)
38. \( -4 = x - 3 \)
39. \( 9 + x = 4 \)
40. \( x + 29 = -29 \)
41. \( 4 + x = -9 \)
42. \( 9 = x - 3 \)
43. \( 7 + r = -23 \)
44. \( a - 5 = -9 \)
Identify reciprocals.

Use the multiplication property to solve equations.

Solve equations of the form \(-x = a\).

Do some steps mentally when solving equations.

### Problem Solving

69. Do you think the equation \(x + 1 = x + 2\) has a real number as a solution? Explain. (We will discuss equations like this in Section 2.5.)

70. Do you think the equation \(x + 4 = x + 4\) has more than one real number as a solution? If so, how many solutions does it have? Explain. (We will discuss equations like this in Section 2.5.)

### Challenge Problems

We can solve equations that contain unknown symbols. Solve each equation for the symbol indicated by adding (or subtracting) a symbol to (or from) both sides of the equation. Explain each answer. (Remember that to solve the equation you want to isolate the symbol you are solving for on one side of the equation.)

71. \(\triangle = \Box\), for \(x\)

72. \(\Box + \triangle = \Box\), for \(\Box\)

73. \(\Box = \Box + \triangle\), for \(\Box\)

74. \(\Box = \triangle + \Box\), for \(\Box\)

### Group Activity

Discuss and answer Exercise 75 as a group.

75. Consider the equation \(2(x + 3) = 2x + 6\).

   a) Group member 1: Determine whether 4 is a solution to the equation.
   
   b) Group member 2: Determine whether \(-2\) is a solution to the equation.
   
   c) Group member 3: Determine whether 0.3 is a solution to the equation.

   d) Each group member: Select a number not used in parts a)–c) and determine whether that number is a solution to the equation.

   e) As a group, write what you think is the solution to the equation \(2(x + 3) = 2x + 6\) and write a paragraph explaining your answer.

### Cumulative Review Exercises


76. \(-\frac{7}{15} + \frac{5}{6}\)

77. \(-\frac{11}{12} + \left(-\frac{3}{8}\right)\)

[2.1] Simplify.

78. \(4x + 3(x - 2) - 5x - 7\)

79. \(-(2r + 4) + 3(4r - 5) - 3t\)

### 2.3 The Multiplication Property of Equality

1. Identify Reciprocals

In Section 1.10, we introduced the reciprocal (or multiplicative inverse) of a number. Recall that two numbers are reciprocals of each other when their product is 1. Some examples of numbers and their reciprocals follow.

<table>
<thead>
<tr>
<th>Number</th>
<th>Reciprocal</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(\frac{1}{2})</td>
<td>((2)\left(\frac{1}{2}\right) = 1)</td>
</tr>
<tr>
<td>(-\frac{3}{5})</td>
<td>(\frac{5}{3})</td>
<td>(\left(-\frac{3}{5}\right)\left(\frac{5}{3}\right) = 1)</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
<td>((-1)(-1) = 1)</td>
</tr>
</tbody>
</table>

The reciprocal of a positive number is a positive number and the reciprocal of a negative number is a negative number. Note that 0 has no reciprocal. Why?
In general, if $a$ represents any nonzero number, its reciprocal is $\frac{1}{a}$. For example, the reciprocal of 3 is $\frac{1}{3}$ and the reciprocal of $-2$ is $\frac{1}{-2}$ or $-\frac{1}{2}$. The reciprocal of $-\frac{3}{5}$ is $\frac{-3}{5}$, which can be written as $1 \div \left( -\frac{3}{5} \right)$. Simplifying, we get \( \left( \frac{1}{1} \right) \left( -\frac{5}{3} \right) = -\frac{5}{3} \). Thus, the reciprocal of $-\frac{3}{5}$ is $-\frac{5}{3}$.

2 Use the Multiplication Property to Solve Equations

In Section 2.2, we used the addition property of equality to solve equations of the form $x + a = b$, where $a$ and $b$ represent real numbers. In this section, we use the multiplication property of equality to solve equations of the form $ax = b$, where $a$ and $b$ represent real numbers.

It is important that you recognize the difference between equations like $x + 2 = 8$ and $2x = 8$. In $x + 2 = 8$, the 2 is a term that is being added to $x$, so we use the addition property to solve the equation. In $2x = 8$, the 2 is a factor of $2x$. The 2 is the coefficient multiplying the $x$, so we use the multiplication property to solve the equation. The multiplication property of equality is used to solve linear equations where the coefficient of the $x$-term is a number other than 1.

Now we present the multiplication property of equality.

**Multiplication Property of Equality**

If $a = b$, then $a \cdot c = b \cdot c$ for any real numbers $a$, $b$, and $c$.

The multiplication property means that both sides of an equation can be multiplied by the same nonzero number without changing the solution. The multiplication property can be used to solve equations of the form $ax = b$. We can isolate the variable in equations of this form by multiplying both sides of the equation by the reciprocal of $a$, which is $\frac{1}{a}$. Doing so makes the numerical coefficient of the variable, $x$, become 1, which can be omitted when we write the variable.

**To Solve, Use the Multiplication Property to Change**

<table>
<thead>
<tr>
<th>Equation</th>
<th>To Solve, Use the Multiplication Property to Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4x = 9$</td>
<td>$4x$ to $1x$</td>
</tr>
<tr>
<td>$-5x = 20$</td>
<td>$-5x$ to $1x$</td>
</tr>
<tr>
<td>$15 = \frac{1}{2}x$</td>
<td>$\frac{1}{2}x$ to $1x$</td>
</tr>
<tr>
<td>$7 = -9x$</td>
<td>$-9x$ to $1x$</td>
</tr>
</tbody>
</table>

Now let’s work some examples.

**EXAMPLE 1** Solve the equation $9x = 63$.

**Solution** To isolate the variable, $x$, we must change the $9x$ on the left side of the equal sign to $1x$. To do this, we multiply both sides of the equation by the reciprocal of 9, which is $\frac{1}{9}$.

\[
9x = 63
\]
\[
\frac{1}{9} \cdot 9x = \frac{1}{9} \cdot 63
\]
\[
\frac{1}{9} \cdot 9x = \frac{1}{9} \cdot 63
\]
\[
\frac{1}{9} \cdot 9x = \frac{1}{9} \cdot 63
\]
\[
\frac{1}{9} \cdot 9x = \frac{1}{9} \cdot 63
\]
\[
1x = 7
\]
\[
x = 7
\]

> Now Try Exercise 9
Notice in Example 1 that \( \frac{1}{9}x \) is replaced by \( x \) in the last step. Usually we do this step mentally.

**EXAMPLE 2**   Solve the equation \( \frac{x}{2} = 4 \).

**Solution**   Since dividing by 2 is the same as multiplying by \( \frac{1}{2} \), the equation \( \frac{x}{2} = 4 \) is the same as \( \frac{1}{2}x = 4 \). We will therefore multiply both sides of the equation by the reciprocal of \( \frac{1}{2} \) which is 2.

\[
\frac{x}{2} = 4 \\
2 \left( \frac{x}{2} \right) = 2 \cdot 4 \quad \text{Multiply both sides by 2.} \\
x = 2 \cdot 4 \\
x = 8
\]

**EXAMPLE 3**   Solve the equation \( \frac{2}{3}x = 6 \).

**Solution**   The reciprocal of \( \frac{2}{3} \) is \( \frac{3}{2} \). We multiply both sides of the equation by \( \frac{3}{2} \).

\[
\frac{2}{3}x = 6 \\
\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 6 \quad \text{Multiply both sides by} \quad \frac{3}{2} \\
x = 9 \\
x = 9
\]

We will show a check of this solution.

Check:

\[
\frac{2}{3}x = 6 \\
\frac{2}{3} \left( \frac{9}{1} \right) = 6 \\
6 = 6 \quad \text{True}
\]

In Example 1, we multiplied both sides of the equation \( 9x = 63 \) by \( \frac{1}{9} \) to isolate the variable. We could have also isolated the variable by dividing both sides of the equation by 9, as follows:

\[
9x = 63 \\
\frac{1}{9} \cdot 9x = \frac{1}{9} \cdot 63 \\
x = 7
\]

We can do this because dividing by 9 is equivalent to multiplying by \( \frac{1}{9} \). Since division can be defined in terms of multiplication \( \left( \frac{a}{b} \right) \text{ means } a \cdot \frac{1}{b} \), the multiplication property also allows us to divide both sides of an equation by the same nonzero number. This process is illustrated in Examples 4 through 6.
EXAMPLE 4  Solve the equation \(8w = 3\).

Solution  In this equation, \(w\) is the variable. To solve the equation we divide both sides of the equation by 8.

\[
\begin{align*}
8w &= 3 \\
\frac{8w}{8} &= \frac{3}{8} \\
w &= \frac{3}{8}
\end{align*}
\]

Now Try Exercise 33

EXAMPLE 5  Solve the equation \(-15 = -3z\).

Solution  In this equation, the variable, \(z\), is on the right side of the equal sign. To isolate \(z\), we divide both sides of the equation by \(-3\).

\[
\begin{align*}
-15 &= -3z \\
\frac{-15}{-3} &= \frac{-3z}{-3} \\
5 &= z
\end{align*}
\]

Now Try Exercise 21

EXAMPLE 6  Solve the equation \(0.24x = 1.20\).

Solution  We begin by dividing both sides of the equation by 0.24 to isolate the variable \(x\).

\[
\begin{align*}
0.24x &= 1.20 \\
\frac{0.24x}{0.24} &= \frac{1.20}{0.24} \\
x &= 5
\end{align*}
\]

Now Try Exercise 35

Helpful Hint

When solving an equation of the form \(ax = b\), we can isolate the variable by

1. multiplying both sides of the equation by the reciprocal of \(a\), \(\frac{1}{a}\), as was done in Examples 1, 2, and 3, or

2. dividing both sides of the equation by \(a\), as was done in Examples 4, 5, and 6.

Either method may be used to isolate the variable. However, if the equation contains a fraction, or fractions, you will arrive at a solution more quickly by multiplying by the reciprocal of \(a\). This is illustrated in Examples 7 and 8.
EXAMPLE 7 \> Solve the equation \(-2x = \frac{3}{5}\).
\begin{align*}
\text{Solution} & \quad \text{Since this equation contains a fraction, we will isolate the variable by multiplying both sides of the equation by } \frac{-1}{2}, \text{ which is the reciprocal of } -2. \\
-2x & = \frac{3}{5} \\
\left(\frac{-1}{2}\right) & \left(-2x\right) = \left(\frac{-1}{2}\right) \left(\frac{3}{5}\right) \quad \text{Multiply both sides by } \frac{-1}{2} \\
1x & = \left(\frac{-1}{2}\right) \left(\frac{3}{5}\right) \\
x & = \frac{-3}{10}
\end{align*}
\> Now Try Exercise 39

In Example 7, if you wished to solve the equation by dividing both sides of the equation by \(-2\), you would have to divide the fraction \(\frac{3}{5}\) by \(-2\).

EXAMPLE 8 \> Solve the equation \(-6 = -\frac{3}{5}x\).
\begin{align*}
\text{Solution} & \quad \text{Since this equation contains a fraction, we will isolate the variable by multiplying both sides of the equation by the reciprocal of } -\frac{3}{5}, \text{ which is } -\frac{5}{3}. \\
-6 & = -\frac{3}{5}x \\
\left(-\frac{5}{3}\right) & \left(-6\right) = \left(-\frac{5}{3}\right) \left(-\frac{3}{5}x\right) \quad \text{Multiply both sides by } -\frac{5}{3} \\
10 & = 1x \\
10 & = x
\end{align*}
\> Now Try Exercise 57

In Example 8, the equation was written as \(-6 = -\frac{3}{5}x\). This equation is equivalent to the equations \(-6 = -\frac{3}{5}x\) and \(-6 = -\frac{3}{5}x\). Can you explain why? All three equations have the same solution, 10.

3 \> Solve Equations of the Form \(-x = a\)

When solving an equation, we may obtain an equation like \(-x = 7\). This is not a solution since \(-x = 7\) means \(-1x = 7\). The solution to an equation is of the form \(x = \text{some number}\). When an equation is of the form \(-x = 7\), we can solve for \(x\) by multiplying both sides of the equation by \(-1\), as illustrated in the following example.

EXAMPLE 9 \> Solve the equation \(-x = 7\).
\begin{align*}
\text{Solution} & \quad -x = 7 \quad \text{means that } -1x = 7. \quad \text{We are solving for } x, \text{ not } -x. \quad \text{We can multiply both sides of the equation by } -1 \text{ to isolate } x \text{ on the left side of the equation.} \\
-x & = 7 \\
-1x & = 7 \\
\left(-1\right) & \left(-1x\right) = \left(-1\right) \left(7\right) \quad \text{Multiply both sides by } -1. \\
1x & = -7 \\
x & = -7
\end{align*}

Check:
\begin{align*}
-x & = 7 \\
-\left(-7\right) & \neq 7 \\
7 & = 7 \quad \text{True}
\end{align*}

Thus, the solution is \(-7\). \> Now Try Exercise 23
Example 9 may also be solved by dividing both sides of the equation by \(-1\). Try this now and see that you get the same solution. Whenever we have the opposite (or negative) of a variable equal to a quantity, as in Example 9, we can solve for the variable by multiplying (or dividing) both sides of the equation by \(-1\).

**EXAMPLE 10**  Solve the equation \(-x = -5\).

**Solution**

\[
\begin{align*}
-1x &= -5 \\
1x &= 5 \\
x &= 5
\end{align*}
\]

*Helpful Hint*

For any real number \(a\), if \(-x = a\), then \(x = -a\).

**Examples**

\[
\begin{align*}
-x &= 7 & -x &= -2 \\
x &= -7 & x &= -(-2) \\
& & x &= 2
\end{align*}
\]

4 Do Some Steps Mentally When Solving Equations

When you feel comfortable using the multiplication property, you may wish to do some of the steps mentally to reduce some of the written work. Now we present two examples worked out in detail, along with their shortened form.

**EXAMPLE 11**  Solve the equation \(-3x = -21\).

**Solution**

\[
\begin{align*}
-3x &= -21 \\
-x &= 7 \\
\frac{3}{-3} &= 7 \\
x &= 7
\end{align*}
\]

*Shortened Form*

\[
\begin{align*}
-3x &= -21 \\
x &= 7
\end{align*}
\]

*Now Try Exercise 61*

**EXAMPLE 12**  Solve the equation \(\frac{1}{5}x = 20\).

**Solution**

\[
\begin{align*}
\frac{1}{5}x &= 20 \\
5 \left( \frac{1}{5}x \right) &= 5(20) \\
x &= 5(20) \\
x &= 100
\end{align*}
\]

*Shortened Form*

\[
\begin{align*}
\frac{1}{5}x &= 20 \\
x &= 5(20) \\
x &= 100
\end{align*}
\]

*Now Try Exercise 63*

In Section 2.2, we discussed the addition property and in this section we discussed the multiplication property. It is important that you understand the difference between the two. The following Helpful Hint should be studied carefully.
Chapter 2  Solving Linear Equations and Inequalities

EXERCISE SET 2.3

Concept/Writing Exercises

1. Explain the multiplication property of equality.
2. Explain why the multiplication property allows us to divide both sides of an equation by a nonzero quantity.
3. a) If where \( a \) represents any real number, what does \( x \) equal?
   b) If \( x = 5 \), what is \( x \)?
   c) If \( -x = -5 \), what is \( x \)?
4. When solving the equation \(-2x = 5\), would you divide both sides of the equation by \(-2\) or by \(5\)? Explain.
5. When solving the equation \(3x = 5\), would you divide both sides of the equation by \(3\) or by \(5\)? Explain.
6. When solving the equation \(4 = \frac{x}{3}\), what would you do to isolate the variable? Explain.
7. When solving the equation \(\frac{x}{2} = 3\), what would you do to isolate the variable? Explain.
8. When solving the equation \(ax = b\) for \(x\), would you divide both sides of the equation by \(a\) or \(b\)? Explain.

Practice the Skills
Solve each equation and check your solution.

9. \(4x = 12\)
10. \(5x = 50\)
11. \(\frac{x}{3} = 7\)
12. \(\frac{y}{5} = 3\)
13. \(-4x = 12\)
14. \(8 = 16y\)
15. \(\frac{x}{4} = -2\)
16. \(\frac{x}{3} = -3\)
17. \(\frac{x}{5} = 1\)
18. \(-7x = 49\)
19. \(-27n = 81\)
20. \(\frac{x}{8} = -3\)
21. \(-7 = 3r\)
22. \(16 = -4y\)
23. \(-x = 13\)
24. \(-x = 9\)
25. \(-x = -8\)
26. \(-x = -15\)
27. \(-\frac{w}{3} = -13\)
28. \(-4 = \frac{c}{7}\)
29. \(4 = -12x\)
30. \(12y = -15\)
31. \(-\frac{x}{3} = -2\)
32. \(-\frac{a}{8} = -7\)
33. \(43t = 26\)
34. \(-24x = -18\)
35. \(-4.2x = -8.4\)
36. \(-3.88 = 1.94y\)
37. \(3x = \frac{3}{5}\)
38. \(7x = -7\)
39. \(5x = -\frac{3}{8}\)
40. \(-2b = -\frac{4}{5}\)
41. \(15 = -\frac{x}{4}\)
42. \(\frac{c}{9} = 0\)
43. \(-\frac{b}{4} = -60\)
44. \(-x = -\frac{5}{9}\)
45. \(\frac{x}{5} = -7\)
46. \(-3r = 0\)
47. \(5 = \frac{x}{4}\)
48. \(-3 = \frac{x}{-5}\)
49. \(\frac{3}{5}d = -30\)
50. \(\frac{2}{7}x = 7\)
51. \(\frac{y}{2} = 0\)
52. \(-6x = \frac{5}{2}\)
Section 2.4 Solving Linear Equations with a Variable on Only One Side of the Equation

53. \( \frac{-7}{8} w = 0 \)  
54. \( -x = \frac{5}{8} \)  
55. \( \frac{1}{5} x = 4.5 \)  
56. \( -\frac{1}{4} x = \frac{3}{4} \)

57. \( -4 = -\frac{2}{3} z \)  
58. \( -9 = \frac{-5}{3} n \)  
59. \( -1.4x = 28.28 \)  
60. \( -0.42x = -2.142 \)

Solve each equation by doing some steps mentally. Check your solution.

61. \( -8x = -56 \)  
62. \( -9x = -45 \)  
63. \( \frac{2}{3} x = 6 \)  
64. \( \frac{1}{3} x = 15 \)

Problem Solving

65. a) Explain the difference between \( 5 + x = 10 \) and \( 5x = 10 \).
   b) Solve \( 5 + x = 10 \).
   c) Solve \( 5x = 10 \).

66. a) Explain the difference between \( 3 + x = 6 \) and \( 3x = 6 \).
   b) Solve \( 3 + x = 6 \).
   c) Solve \( 3x = 6 \).

67. Consider the equation \( \frac{2}{3} x = 4 \). This equation could be solved by multiplying both sides of the equation by \( \frac{3}{2} \), the reciprocal of \( \frac{2}{3} \), or by dividing both sides of the equation by \( \frac{2}{3} \). Which method do you feel would be easier? Explain your answer. Find the solution to the equation.

68. Consider the equation \( 4x = \frac{3}{5} \). Would it be easier to solve this equation by dividing both sides of the equation by 4 or by multiplying both sides of the equation by \( \frac{1}{4} \), the reciprocal of 4? Explain your answer. Find the solution to the problem.

69. Consider the equation \( \frac{3}{7} x = \frac{4}{5} \). Would it be easier to solve this equation by dividing both sides of the equation by \( \frac{3}{7} \) or by multiplying both sides of the equation by \( \frac{7}{3} \), the reciprocal of \( \frac{3}{7} \)? Explain your answer. Find the solution to the equation.

Challenge Problems

70. Consider the equation \( \square \cdot \triangle = \triangle \).
   a) To solve for \( \square \), what symbol do we need to isolate?
   b) How would you isolate the symbol you specified in part a)?
   c) Solve the equation for \( \square \).

71. Consider the equation \( \otimes = \triangle \).
   a) To solve for \( \otimes \), what symbol do we need to isolate?
   b) How would you isolate the symbol you specified in part a)?
   c) Solve the equation for \( \otimes \).

Cumulative Review Exercises

[1.7] 73. Subtract \(-4\) from \(-8\).
[1.8] 74. Evaluate \((-3)(-2)(5)(-1)\).
[1.9] 75. Evaluate \(4^2 - 2^3 + 6 + 3 + 6\).
[1.10] 76. Name the property illustrated.
   \[ 2 + (4 + y) = (2 + 4) + y \]
[2.2] 77. Solve the equation \(-48 = x + 9\).

2.4 Solving Linear Equations with a Variable on Only One Side of the Equation

1. Solve linear equations with a variable on only one side of the equal sign.
2. Solve equations containing decimal numbers or fractions.

1. Solve Linear Equations with a Variable on Only One Side of the Equal Sign

In this section, we discuss how to solve linear equations using both the addition and multiplication properties of equality when a variable appears on only one side of the equal sign. In Section 2.5, we will discuss how to solve linear equations using both properties when a variable appears on both sides of the equal sign.

The general procedure we use to solve equations is to “isolate the variable.” That is, get the variable alone on one side of the equal sign.
No one method is the “best” to solve all linear equations. But the following general procedure can be used to solve linear equations when the variable appears on only one side of the equation.

1. If the equation contains fractions, multiply both sides of the equation by the least common denominator (LCD). This will eliminate the fractions from the equation.
2. Use the distributive property to remove parentheses.
3. Combine like terms on the same side of the equal sign.
4. Use the addition property to obtain an equation with the term containing the variable on one side of the equal sign and a constant on the other side. This will result in an equation of the form \( ax = b \).
5. Use the multiplication property to isolate the variable. This will give a solution of the form \( x = \frac{b}{a} \) (or \( 1x = \frac{b}{a} \)).
6. Check the solution in the original equation.

When solving an equation, you should always check your solution, as is indicated in step 6. To conserve space, we will not show all checks.

**When solving an equation, remember that our goal is to isolate the variable on one side of the equation.**

Consider the equation \( 2x + 4 = 10 \) which contains no fractions or parentheses, and no like terms on the same side of the equal sign. Therefore, we start with step 4, using the addition property. Remember that the addition property allows us to add (or subtract) the same quantity to (or from) both sides of an equation without changing its solution. Here we subtract 4 from both sides of the equation to get the \( 2x \) by itself on one side of the equal sign.

\[
\begin{align*}
\text{Equation} \\
2x + 4 &= 10 \\
2x + 4 - 4 &= 10 - 4 \\
2x + 0 &= 6 \\
\text{or} \quad 2x &= 6 \\
\text{Addition property} \\
\text{x term is now isolated}
\end{align*}
\]

Notice how the term containing the variable, \( 2x \), is now by itself on one side of the equal sign. We can now say that we have isolated the term containing the variable or have isolated the variable term. Now we use the multiplication property, step 5, to isolate the variable, \( x \). Remember that the multiplication property allows us to multiply or divide both sides of the equation by the same nonzero number without changing its solution. Here we divide both sides of the equation by 2, the coefficient of the term containing the variable, to obtain the solution, 3.

\[
\begin{align*}
2x &= 6 \\
\frac{2x}{2} &= \frac{6}{2} \\
\text{Multiplication property} \\
1x &= 3 \\
x &= 3 \\
\text{x is now isolated}
\end{align*}
\]

The solution to the equation \( 2x + 4 = 10 \) is 3. Now let’s work some examples.
Section 2.4 Solving Linear Equations with a Variable on Only One Side of the Equation

**EXAMPLE 1**  Solve the equation $5x - 7 = 13$.

*Solution* We will follow the procedure outlined for solving equations. Since the equation contains no fractions nor parentheses, and since there are no like terms to be combined, we start with step 4.

\[
\begin{align*}
5x - 7 &= 13 \\
\text{Step 4} & \quad 5x - 7 + 7 = 13 + 7 \quad \text{Add 7 to both sides.} \\
5x &= 20 \\
\text{Step 5} & \quad \frac{5x}{5} = \frac{20}{5} \quad \text{Divide both sides by 5.} \\
x &= 4 \\
\end{align*}
\]

**Step 6 Check:**

\[
\begin{align*}
5x - 7 &= 13 \\
5(4) - 7 &= 13 \\
20 - 7 &= 13 \\
13 &= 13 \quad \text{True}
\end{align*}
\]

Since the check is true, the solution is 4. Note that after completing step 4, we obtain $5x = 20$, which is an equation of the form $ax = b$. After completing step 5, we obtain the answer in the form $x = \text{some real number}$.  

» Now Try Exercise 15

**Helpful Hint**

When solving an equation that does not contain fractions, the *addition property* (step 4) is to be used before the *multiplication property* (step 5). If you use the multiplication property before the addition property, it is still possible to obtain the correct answer. However, you will usually have to do more work, and you may end up working with fractions. What would happen if you tried to solve Example 1 using the multiplication property before the addition property?

**EXAMPLE 2**  Solve the equation $-2r - 6 = -3$.

*Solution*

\[
\begin{align*}
-2r - 6 &= -3 \\
\text{Step 4} & \quad -2r - 6 + 6 = -3 + 6 \quad \text{Add 6 to both sides.} \\
-2r &= 3 \\
\text{Step 5} & \quad -\frac{2r}{-2} = \frac{3}{-2} \quad \text{Divide both sides by \(-2\).} \\
r &= -\frac{3}{2} \\
\end{align*}
\]

**Step 6 Check:**

\[
\begin{align*}
-2r - 6 &= -3 \\
-2\left(-\frac{3}{2}\right) - 6 &= -3 \\
3 - 6 &= -3 \\
-3 &= -3 \quad \text{True}
\end{align*}
\]

The solution is $-\frac{3}{2}$.  

» Now Try Exercise 23

Note that checks are always made with the *original* equation. In some of the following examples, the check will be omitted to save space. You should check all of your answers.
EXAMPLE 3 ▶ Solve the equation $16 = 4x + 6 - 2x$.

Solution ▶ Again we must isolate the variable, $x$. Since the right side of the equation has two like terms containing the variable, $x$, we will first combine these like terms.

\[ 16 = 4x + 6 - 2x \]

**Step 3**

\[ 16 = 2x + 6 \]

Like terms were combined.

**Step 4**

\[ 16 - 6 = 2x + 6 - 6 \]

10 = 2x

Subtract 6 from both sides.

**Step 5**

\[ \frac{10}{2} = \frac{2x}{2} \]

5 = x

Divide both sides by 2.

▷ Now Try Exercise 37

The preceding solution can be condensed as follows.

\[ 16 = 4x + 6 - 2x \]

16 = 2x + 6 ▶ Like terms were combined.

10 = 2x ▶ 6 was subtracted from both sides.

5 = x ▶ Both sides were divided by 2.

EXAMPLE 4 ▶ Solve the equation $5x - 2(x + 4) = 3$.

Solution ▶

\[ 5x - 2(x + 4) = 3 \]

**Step 2**

\[ 5x - 2x - 8 = 3 \]

Distributive property was used.

**Step 3**

\[ 3x - 8 = 3 \]

Like terms were combined.

**Step 4**

\[ 3x - 8 + 8 = 3 + 8 \]

Add 8 to both sides.

\[ 3x = 11 \]

\[ \frac{3x}{3} = \frac{11}{3} \]

\[ x = \frac{11}{3} \]

Divide both sides by 3.

▷ Now Try Exercise 65

The solution to Example 4 can be condensed as follows:

\[ 5x - 2(x + 4) = 3 \]

\[ 5x - 2x - 8 = 3 \]

Distributive property was used.

\[ 3x - 8 = 3 \]

Like terms were combined.

\[ 3x = 11 \]

\[ x = \frac{11}{3} \]

Both sides were divided by 3.

EXAMPLE 5 ▶ Solve the equation $3p - (2p + 5) = 7$.

Solution ▶

\[ 3p - (2p + 5) = 7 \]

\[ 3p - 2p - 5 = 7 \]

Distributive property was used.

\[ p - 5 = 7 \]

Like terms were combined.

\[ p = 12 \]

5 was added to both sides.

▷ Now Try Exercise 69
Section 2.4 Solving Linear Equations with a Variable on Only One Side of the Equation

2 Solve Equations Containing Decimal Numbers or Fractions

In Chapter 3, we will be solving many equations that contain decimal numbers. To solve such equations, we may follow the same procedure as outlined earlier. Example 6 illustrates two methods to solve an equation that contains decimal numbers.

EXAMPLE 6 Solve the equation $x + 1.24 - 0.07x = 4.96$.

Solution We will work this example using two methods. In method 1, we work with decimal numbers throughout the solving process. In method 2, we multiply both sides of the equation by a power of 10 to change the decimal numbers to whole numbers.

Method 1

\[
\begin{align*}
&x + 1.24 - 0.07x = 4.96 \\
&0.93x + 1.24 = 4.96 \\
&0.93x + 1.24 - 1.24 = 4.96 - 1.24 \\
&0.93x = 3.72 \\
&x = 4
\end{align*}
\]

Method 2 Some students prefer to eliminate the decimal numbers from the equation by multiplying both sides of the equation by 10 if the decimal numbers are given in tenths, by 100 if the decimal numbers are given in hundredths, and so on. In Example 6, since the decimal numbers are in hundredths, you can eliminate the decimals from the equation by multiplying both sides of the equation by 100. This alternate method would give the following.

\[
\begin{align*}
&x + 1.24 - 0.07x = 4.96 \\
&100(x + 1.24 - 0.07x) = 100(4.96) \\
&100x + 124 - 7x = 496 \\
&93x + 124 = 496 \\
&93x = 372 \\
&x = 4
\end{align*}
\]

Study both methods provided to see which method you prefer.
EXAMPLE 7 • Solve $\frac{1}{5}(x + 1) = 1.$

**Solution** The LCD of the fraction is 5. We will begin by multiplying both sides of the equation by the LCD. This step will eliminate fractions from the equation.

\[
\frac{1}{5}(x + 1) = 1
\]

**Step 1**

\[5 \left( \frac{1}{5}(x + 1) \right) = 5 \cdot 1 \quad \text{Multiply both sides by the LCD, 5.} \]

\[5 \left( \frac{1}{5} \right)(x + 1) = 5 \]

\[x + 1 = 5 \]

**Step 4**

\[x = 4 \quad \text{1 was subtracted from both sides.} \]

**Step 6** Check:

\[
\frac{1}{5}(x + 1) = 1
\]

\[\frac{1}{5}(4 + 1) = 1 \]

\[\frac{1}{5}(5) = 1 \]

\[1 = 1 \quad \text{True} \]

The solution is 4.  

Now Try Exercise 45

Example 7 could also be written as \(\frac{x + 1}{5} = 1.\) To solve this equation, we would begin by multiplying both sides of the equation by the LCD, 5, as follows.

\[
\frac{x + 1}{5} = 1
\]

\[5 \left( \frac{x + 1}{5} \right) = 5 \cdot 1 \]

\[x + 1 = 5 \]

\[x = 4 \]

EXAMPLE 8 • Solve the equation $\frac{d}{2} + 3d = 14.$

**Solution** Step 1 tells us to multiply both sides of the equation by the LCD, 2. This step will eliminate fractions from the equation.

**Step 1**

\[2 \left( \frac{d}{2} + 3d \right) = 2 \cdot 14 \quad \text{Multiply both sides by the LCD, 2.} \]

**Step 2**

\[2 \left( \frac{d}{2} \right) + 2 \cdot 3d = 2 \cdot 14 \quad \text{Distributive property} \]

\[d + 6d = 28 \]

**Step 3**

\[7d = 28 \quad \text{Like terms were combined.} \]

**Step 5**

\[d = 4 \quad \text{Both sides were divided by 7.} \]

**Step 6** Check:

\[\frac{d}{2} + 3d = 14 \]

\[\frac{4}{2} + 3(4) = 14 \]

\[2 + 12 = 14 \]

\[14 = 14 \quad \text{True} \]

Now Try Exercise 89
EXAMPLE 9\[ 
\text{Solve the equation } \frac{1}{5}x - \frac{3}{8}x = \frac{1}{10}. 
\]

**Solution** The LCD of 5, 8, and 10 is 40. Multiply both sides of the equation by 40 to eliminate fractions from the equation.

\[
\begin{align*}
\text{Step 1} & \quad 40 \left( \frac{1}{5}x - \frac{3}{8}x \right) = 40 \left( \frac{1}{10} \right) \\
& \quad \text{Multiply both sides by the LCD, 40.} \\
\text{Step 2} & \quad 40 \left( \frac{1}{5}x \right) - 40 \left( \frac{3}{8}x \right) = 40 \left( \frac{1}{10} \right) \\
& \quad \text{Distributive property} \\
& \quad 8x - 15x = 4 \\
& \quad \text{Like terms were combined.} \\
\text{Step 3} & \quad -7x = 4 \\
\text{Step 4} & \quad x = -\frac{4}{7} \\
& \quad \text{Both sides were divided by } -7. \\
\text{Step 5} & \quad \text{Check:} \\
& \quad \frac{1}{5}x - \frac{3}{8}x = \frac{1}{10} \\
& \quad \frac{1}{5} \left( \frac{-4}{7} \right) - \frac{3}{8} \left( \frac{-4}{7} \right) = \frac{1}{10} \\
& \quad \text{Substitute } -\frac{4}{7} \text{ for each } x. \\
& \quad -\frac{4}{35} + \frac{3}{14} = \frac{1}{10} \\
& \quad \text{Divide out common factors, then multiply fractions.} \\
& \quad -\frac{8}{70} + \frac{15}{70} = \frac{7}{70} \\
& \quad \text{Write each fraction with the LCD, 70.} \\
& \quad \frac{7}{70} = \frac{7}{70} \\
& \quad \text{True} \\
\end{align*}
\]

**Helpful Hint**

In Example 9, we multiplied both sides of the equation by the LCD, 40. When solving equations containing fractions, multiplying both sides of the equation by any common denominator will eventually lead to the correct answer (if you don’t make a mistake), but you may have to work with larger numbers. In Example 9, if you multiplied both sides of the equation by 80, 120, or 160, for example, you would eventually obtain the answer $x = -\frac{4}{7}$. When solving equations containing fractions, you should multiply both sides of the equation by the LCD. But if you mistakenly multiply both sides of the equation by a different common denominator to clear fractions, you will still obtain the correct answer. To show that other common denominators may be used, solve Example 9 now by multiplying both sides of the equation by the common denominator 80 instead of the LCD, 40.

When checking solutions to equations that contain fractions, you may sometimes want to perform the check using a calculator. When checking a solution using a calculator, work with each side of the equation separately. Below we show the steps used to evaluate the left side of the equation in Example 9 for $x = -\frac{4}{7}$ using a scientific calculator.

\[
\frac{1}{5}x - \frac{3}{8}x = \frac{1}{10} \\
\frac{1}{5} \left( -\frac{4}{7} \right) - \frac{3}{8} \left( -\frac{4}{7} \right) = \frac{1}{10}
\]

Evaluate the left side of the equation.

\[
1 \div 5 \times 4 +/− 7 \div 3 \div 8 \times 4 \div/− 7 = 0.1
\]

Since the right side of the equation $\frac{1}{10} = 0.1$, the answer checks.

*Keystrokes may differ on some scientific calculators. Read the instruction manual for your calculator.*

Some of the most commonly used terms in algebra are “evaluate,” “simplify,” “solve,” and “check.” Make sure you understand what each term means and when each term is used.

**Evaluate:** To evaluate an expression means to find its numerical value.

Evaluate

\[
16 \div 2^2 + 36 \div 4
= 16 \div 4 + 36 \div 4
= 4 + 36 \div 4
= 4 + 9
= 13
\]

\[
-x^2 + 3x - 2 \text{ when } x = 4
= -4^2 + 3(4) - 2
= -16 + 3(4) - 2
= -16 + 12 - 2
= -4 - 2
= -6
\]

**Simplify:** To simplify an expression means to perform the operations and combine like terms.

Simplify

\[
3(x - 2) - 4(2x + 3)
= 3x - 6 - 8x - 12
= -5x - 18
\]

Note that when you simplify an expression containing variables you do not generally end up with just a numerical value unless all the variable terms happen to add to zero.

**Solve:** To solve an equation means to find the value or the values of the variable that make the equation a true statement.

Solve

\[
2x + 3(x + 1) = 18
\]

\[
x = 3
\]

**Check:** To check the proposed solution to an equation, substitute the value in the original equation. If this substitution results in a true statement, then the answer checks. For example, to check the solution to the equation just solved, we substitute 3 for \(x\) in the original equation.

Check

\[
2x + 3(x + 1) = 18
\]

\[
6 + 12 = 18 \quad \text{True}
\]

Since we obtained a true statement, the 3 checks.

It is important to realize that expressions may be evaluated or simplified (depending on the type of problem) and equations are solved and then checked.
Explain, step-by-step, how to solve the equation \( x + 3 = 2x + 5 \) contain a variable on only one side of the equation? Explain.

2. Does the equation \( 2x - 4 = 3 \) contain a variable on only one side of the equation? Explain.

3. If \( x = \frac{1}{3} \), what does \( x \) equal?

4. If \( x = -\frac{3}{5} \), what does \( x \) equal?

5. If \( -x = \frac{1}{2} \), what does \( x \) equal?

6. If \( -x = \frac{7}{8} \), what does \( x \) equal?

7. If \( -x = -\frac{4}{9} \), what does \( x \) equal?

8. If \( -x = -\frac{3}{5} \), what does \( x \) equal?

Do you evaluate or solve an expression? Explain.

Do you evaluate or solve an equation? Explain.

Write the general procedure for solving an equation where the variable appears on only one side of the equal sign.

When solving equations that contain fractions, what is the first step in the process of solving the equation?

Explain, in a step-by-step manner, how to solve the equation \( 2(3x + 4) = -4 \).

Solve the equation by following the steps you listed in part a).

Explain, step-by-step, how to solve the equation \( 4x - 2(x + 3) = 4 \).

Solve the equation by following the steps you listed in part a).

Practice the Skills

Solve each equation. You may wish to use a calculator to solve equations containing decimal numbers.

15. \( 5x - 6 = 19 \)
16. \( 2x - 4 = 8 \)
17. \( -4w - 5 = 11 \)
18. \( -4x + 6 = 20 \)
19. \( 3x + 6 = 12 \)
20. \( 6 - 3x = 18 \)
21. \( 5x - 2 = 10 \)
22. \( -2r + 9 = 21 \)
23. \( -5k - 4 = -19 \)
24. \( -4x - 7 = -6 \)
25. \( 12 - x = 9 \)
26. \( -3x - 3 = -12 \)
27. \( 8 + 3x = 19 \)
28. \( -2x + 7 = -10 \)
29. \( 16x + 5 = -14 \)
30. \( 19 = 25 + 4x \)
31. \( -4.2 = 3x + 25.8 \)
32. \( -24 + 16x = -24 \)
33. \( 7r - 16 = -2 \)
34. \( -2w + 4 = -8 \)
35. \( 60 = -5x + 9 \)
36. \( 15 = 7x + 1 \)
37. \( 14 = 5x - 8 - 3x \)
38. \( 15 = 6x - 3 + 3x \)
39. \( 2.3x - 9.34 = 6.3 \)
40. \( x + 0.05x = 21 \)
41. \( 0.01y + 2.25 - 0.01y = 5.85 \)
42. \( 0.15 = 0.05x - 1.35 - 0.20x \)
43. \( 28.8 = x + 1.40x \)
44. \( 8.40 = 2.45x - 1.05x \)
45. \( \frac{1}{2}(x + 6) = 4 \)
46. \( m - \frac{6}{5} = 2 \)
47. \( d + \frac{3}{7} = 9 \)
48. \( \frac{1}{3}(x + 2) = -3 \)
49. \( \frac{1}{3}(r - 5) = -6 \)
50. \( \frac{2}{3}(n - 3) = 8 \)
51. \( \frac{3}{4}(x - 5) = -12 \)
52. \( \frac{1}{4} = \frac{z + 1}{4} \)
53. \( \frac{x + 4}{7} = \frac{3}{7} \)
54. \( \frac{4x + 5}{6} = \frac{7}{2} \)
55. \( \frac{3}{4} = \frac{4m - 5}{6} \)
56. \( \frac{5}{6} = \frac{5t - 4}{2} \)
57. \( 4(n + 2) = 8 \)
58. \( 3(x - 2) = 12 \)
59. \( -2(x - 3) = 26 \)
60. \( 5(3 - x) = 15 \)
61. \( -4 = -(x + 5) \)
62. \( -3(2 - 3x) = 9 \)
63. \( 12 = 4(x - 3) \)
64. \( -2(x + 8) - 5 = 1 \)
65. \( 2x - 3(x + 5) = 6 \)
66. \( 5(3x + 1) - 12x = -2 \)
67. \( -3r - 4(r + 2) = 11 \)
68. \( 9 = -2(a - 3) \)
69. \( x - 3(2x + 3) = 11 \)
70. \( 3y - (y + 5) = 9 \)
71. \( 5x + 3x - 4x - 7 = 9 \)
72. \( 4(x + 2) = 13 \)
73. \( 0.7(x - 3) = 1.4 \)
74. \( 21 + (c - 9) = 24 \)
75. \( 2.5(4q - 3) = 0.5 \)
76. \( 0.1(2.4x + 5) = 1.7 \)
77. \( 3 = 2(x + 3) + 2 = 1 \)
78. \( 2(3x - 4) - 4x = 12 \)
79. \( 1 + (x + 3) + 6x = 6 \)
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81. $4.85 - 6.4x + 1.11 = 22.6$
82. $5.76 - 4.24x - 1.9x = 27.864$
83. $7 = 8 - 5(m + 3)$
84. $4 = \frac{3x + 1}{7}$
85. $10 = \frac{2x + 4}{5}$
86. $12 = \frac{4d - 1}{3}$
87. $x + \frac{2}{3} = \frac{3}{5}$
88. $n - \frac{1}{4} = \frac{1}{2}$
89. $\frac{x}{3} + 2r = 7$
90. $\frac{x}{4} - 6x = 23$
91. $\frac{3}{7} = \frac{3r}{4} + 1$
92. $\frac{5}{8} = \frac{5t}{6} + 2$
93. $\frac{1}{2}r + \frac{1}{3}r = 7$
94. $\frac{x}{3} - \frac{3x}{4} = \frac{1}{12}$
95. $\frac{2}{8} + \frac{3}{4} = \frac{n}{5}$
96. $\frac{x}{4} - \frac{x}{6} = \frac{1}{4}$
97. $\frac{1}{2}x + 4 = \frac{6}{7}$
98. $\frac{4}{5} + n = \frac{1}{3}$
99. $\frac{4}{5} - \frac{3}{4} = \frac{1}{10}$
100. $\frac{1}{3}x - \frac{3}{4}x = \frac{1}{5}$
101. $\frac{4}{9} = \frac{1}{3}(n - 7)$
102. $\frac{-3}{8} = \frac{1}{8} - \frac{2x}{7}$
103. $\frac{-3}{5} = \frac{-1}{9} - \frac{3}{4}$
104. $\frac{-3}{5} = \frac{-1}{6} - \frac{5}{4}m$

Problem Solving

105. a) Explain why it is easier to solve the equation $3x + 2 = 11$ by first subtracting 2 from both sides of the equation rather than by first dividing both sides of the equation by 3.

b) Solve the equation.

106. a) Explain why it is easier to solve the equation $5x - 3 = 12$ by first adding 3 to both sides of the equation rather than by first dividing both sides of the equation by 5.

b) Solve the equation.

Challenge Problems

For exercises 107–109, solve the equation.

107. $3(x - 2) - (x + 5) - 2(3 - 2x) = 18$
108. $-6 = -(x - 5) - 3(5 + 2x) - 4(2x - 4)$
109. $4[3 - 2(x + 4)] - (x + 3) = 13$
110. Solve the equation $\Box - \n = @ for \Box$.

Group Activity

In Chapter 3, we will discuss procedures for writing application problems as equations. Let’s look at an application now.

Birthday Party John Logan purchased 2 large chocolate bars and a birthday card. The birthday card cost $3. The total cost was $9. What was the price of a single chocolate bar?

This problem can be represented by the equation $2x + 3 = 9$, which can be used to solve the problem. Solving the equation we find that $x$, the price of a single chocolate bar, is $3$.

For Exercises 111 and 112, each group member should do parts a) and b). Then do part c) as a group.

a) Obtain an equation that can be used to solve the problem.

b) Solve the equation and answer the question.

c) Compare and check each other's work.

111. Stationery Eduardo Verner purchased three boxes of stationery. He also purchased wrapping paper and thank-you cards. If the wrapping paper and thank-you cards together cost $6, and the total he paid was $42, find the cost of a box of stationery.

112. Candies Mahandi Ison purchased three rolls of peppermint candies and the local newspaper. The newspaper cost 50 cents. He paid $2.75 in all. What did a roll of candies cost?

Cumulative Review Exercises

113. True or false: Every real number is a rational number.

114. Evaluate $[5(2 - 6) + 3(8 + 4)]^2$.

115. To solve an equation, what do you need to do to the variable?

116. To solve the equation $7 = -4x$, would you add 4 to both sides of the equation or divide both sides of the equation by $-4$? Explain your answer.
Section 2.5 Solving Linear Equations with the Variable on Both Sides of the Equation

Mid-Chapter Test: 2.1–2.4

To find out how well you understand the chapter material to this point, take this brief test. The answers, and the section where the material was initially discussed, are given in the back of the book. Review any questions that you answered incorrectly.

1. Solve equations with the variable on both sides of the equal sign.
2. Solve equations containing decimal numbers or fractions.
3. Identify identities and contradictions.

In Exercises 1 and 2, combine like terms.

1. \(5x - 9y - 12 + 4y - 7x + 6\)
2. \(\frac{2}{3}x - 8 - \frac{3}{4}x + \frac{1}{2}\)

In Exercises 3 and 4, use the distributive property to remove parentheses.

3. \(-4(2a - 3b + 6)\)
4. \(1.6(2.1x - 3.4y - 5.2)\)

5. Simplify \(5(t - 3) - 3(t + 7) - 2\).
6. Is \(x = 2\) a solution of \(3(x - 4) = -2(x + 1)\)?
7. Is \(p = \frac{2}{5}\) a solution of \(7p - 3 = 2p - 5\)?

In Exercises 8–10, solve each equation and check your solution.

8. \(x - 5 = -9\)
9. \(12 + x = -4\)
10. \(-16 = 7 + y\)

11. When solving the equation \(\frac{x}{4} = 5\), what would you do to isolate the variable? Explain.

In Exercises 12–15, solve each equation and check your solution.

12. \(6 = 12y\)
13. \(\frac{x}{8} = 3\)
14. \(-\frac{x}{5} = -2\)
15. \(-x = \frac{3}{7}\)

In Exercises 16–20, solve each equation.

16. \(6x - 3 = 12\)
17. \(-4 = -2w - 7\)
18. \(\frac{3}{4} = \frac{4n - 1}{6}\)
19. \(-5(x + 4) - 7 = 3\)
20. \(8 - 9(y + 4) + 6 = -2\)

2.5 Solving Linear Equations with the Variable on Both Sides of the Equation

1. Solve Equations with the Variable on Both Sides of the Equal Sign

The equation \(4x + 6 = 2x + 4\) contains the variable, \(x\), on both sides of the equal sign. To solve equations of this type, we must use the appropriate properties to rewrite the equation with all terms containing the variable on only one side of the equal sign and all terms not containing the variable on the other side of the equal sign. Following is a general procedure, similar to the one outlined in Section 2.4, that can be used to solve linear equations with the variable on both sides of the equal sign. The steps in the procedure are only guidelines to use. For example, there may be times when you may choose to use the distributive property, step 2, before multiplying both sides of the equation by the LCD, step 1. We will illustrate this in Examples 8 and 9.

To Solve Linear Equations with the Variable on Both Sides of the Equal Sign

1. If the equation contains fractions, multiply both sides of the equation by the least common denominator. This will eliminate fractions from the equation.
2. Use the distributive property to remove parentheses.
3. Combine like terms on the same side of the equal sign.
4. Use the addition property to rewrite the equation with all terms containing the variable on one side of the equal sign and all terms not containing the variable on the other side of the equal sign. It may be necessary to use the addition property twice to accomplish this goal. You will eventually get an equation of the form \(ax = b\).
5. Use the multiplication property to isolate the variable. This will give a solution of the form \(x = \text{some number}\).
6. Check the solution in the original equation.
The steps listed on page 129 are basically the same as the steps listed in the boxed procedure on page 120, except that in step 4 you may need to use the addition property more than once to obtain an equation of the form \( ax = b \).

Remember that our goal in solving an equation is to isolate the variable, that is, to get the variable alone on one side of the equation.

Consider the equation \( 3x + 4 = x + 12 \) which contains no fractions or parentheses, and no like terms on the same side of the equal sign. Therefore, we start with step 4, the addition property. We will use the addition property twice in order to obtain an equation where the variable appears on only one side of the equal sign. We begin by subtracting \( x \) from both sides of the equation to get all the terms containing the variable on the left side of the equation. This will give the following:

\[
\begin{align*}
\text{Equation} & & \text{Addition property} \\
3x + 4 &= x + 12 & \quad & 3x - x + 4 = x - x + 12 \\
\text{or} & & \text{Variable appears only on left side of equal sign.} \\
2x + 4 &= 12
\end{align*}
\]

Notice that the variable, \( x \), now appears on only one side of the equation. However, \( +4 \) still appears on the same side of the equal sign as the \( 2x \). We use the addition property a second time to get the term containing the variable by itself on one side of the equation. Subtracting 4 from both sides of the equation gives \( 2x = 8 \), which is an equation of the form \( ax = b \).

\[
\begin{align*}
\text{Equation} & & \text{Addition property} \\
2x + 4 &= 12 & 2x + 4 - 4 &= 12 - 4 \\
\text{or} & & \text{x-term is now isolated.} \\
2x &= 8
\end{align*}
\]

The \( x \)-term, \( 2x \), is now by itself on one side of the equation. Therefore, we have isolated the \( x \)-term on the left side of the equation. We can now use the multiplication property, step 5, to isolate the variable and solve the equation for \( x \). We divide both sides of the equation by 2 to isolate the variable and solve the equation.

\[
\begin{align*}
2x &= 8 & \text{Multiplication property} \\
\frac{2x}{2} &= \frac{8}{2} & \text{x is now isolated.} \\
x &= 4
\end{align*}
\]

The solution to the equation is 4.

**EXAMPLE 1** Solve the equation \( 4x + 6 = 2x + 4 \).

Solution We start by getting all the terms with the variable on one side of the equal sign and all terms without the variable on the other side. The terms with the variable may be collected on either side of the equal sign. Many methods can be used to get the terms with the variable by themselves on one side of the equal sign. We will illustrate two. In method 1, we will collect all terms with the variable on the left side of the equation. In method 2, we will collect all terms with the variable on the right side of the equation. In both methods, we will follow the steps given in the box on page 129. Since this equation does not contain fractions or parentheses, and there are no like terms on the same side of the equal sign, we begin with step 4.

**Method 1**: Isolate the variable term on the left.

\[
4x + 6 = 2x + 4
\]

\[
\text{Step 4} \quad 4x - 2x + 6 = 2x - 2x + 4 \quad \text{Subtract } 2x \text{ from both sides.}
\]

\[
2x + 6 = 4
\]
Section 2.5 Solving Linear Equations with the Variable on Both Sides of the Equation

Step 4 \[ 2x + 6 - 6 = 4 \]
\[ 2x = -2 \]
\[ x = -1 \]

Method 2: Isolate the variable term on the right.
\[ 4x + 6 = 2x + 4 \]
Step 4 \[ 4x - 4x + 6 = 2x - 4x + 4 \]
\[ 6 = -2x + 4 \]
Step 4 \[ 6 - 4 = -2x + 4 \]
\[ 2 = -2x \]
\[ x = -1 \]

Step 5 \[ \frac{2}{-2} = \frac{-2}{-2} \]

The same answer is obtained whether we collect the terms with the variable on the left or right side. However, we need to divide both sides of the equation by a negative number in method 2.

Step 6 Check:
\[ 4(-1) + 6 \neq 2(-1) + 4 \]
\[ -4 + 6 \neq -2 + 4 \]
\[ 2 = 2 \]
True

Since the check is true, the solution is \(-1\).

EXAMPLE 2 \( \times \) Solve the equation \( 2x - 3 - 5x = 13 + 4x - 2 \).

Solution \( \) We will choose to collect the terms containing the variable on the right side of the equation in order to create a positive coefficient of \( x \). Since there are like terms on the same side of the equal sign, we will begin by combining these like terms.

Step 3 \[ 2x - 3 - 5x = 13 + 4x - 2 \]
Like terms were combined.

Step 4 \[ -3x + 3x - 3 = 4x + 3x + 11 \]
Add 3x to both sides.

Step 4 \[ -3 - 11 = 7x + 11 - 11 \]
Subtract 11 from both sides.

Step 5 \[ -14 = 7x \]
Divide both sides by 7.

Step 6 Check:
\[ 2(-2) - 3 - 5(-2) \neq 13 + 4(-2) - 2 \]
\[ -4 - 3 + 10 \neq 13 - 8 - 2 \]
\[ -7 + 10 \neq 5 - 2 \]
\[ 3 = 3 \]
True

Since the check is true, the solution is \(-2\).
Chapter 2 Solving Linear Equations and Inequalities

The solution to Example 2 could be condensed as follows:

\[ 2x - 3 - 5x = 13 + 4x - 2 \]
\[ -3x - 3 = 4x + 11 \]  
\[ -3 = 7x + 11 \]  
\[ -14 = 7x \]  
\[ -2 = x \]  

Like terms were combined.

3x was added to both sides.

11 was subtracted from both sides.

Both sides were divided by 7.

We solved Example 2 by moving the terms containing the variable to the right side of the equation. Now rework the problem by moving the terms containing the variable to the left side of the equation. You should obtain the same answer.

EXAMPLE 3 

Solve the equation \(2(p + 3) = -3p + 10\).

**Solution**

\[ 2(p + 3) = -3p + 10 \]

Step 2

\[ 2p + 6 = -3p + 10 \]

Distributive property was used.

Step 4

\[ 2p + 3p + 6 = -3p + 3p + 10 \]

Add 3p to both sides.

\[ 5p + 6 = 10 \]

Step 4

\[ 5p + 6 - 6 = 10 - 6 \]

Subtract 6 from both sides.

\[ 5p = 4 \]

Step 5

\[ \frac{5p}{5} = \frac{4}{5} \]

Divide both sides by 5.

\[ p = \frac{4}{5} \]

The solution is \(\frac{4}{5}\).

Now Try Exercise 27

The solution to Example 3 could be condensed as follows:

\[ 2(p + 3) = -3p + 10 \]

Distributive property was used.

\[ 2p + 6 = -3p + 10 \]

3p was added to both sides.

\[ 5p + 6 = 10 \]

6 was subtracted from both sides.

\[ 5p = 4 \]

Both sides were divided by 5.

\[ p = \frac{4}{5} \]

**Helpful Hint**

After the distributive property was used in Example 3, we obtained the equation \(2p + 6 = -3p + 10\). Then we had to decide whether to collect terms with the variable on the left or the right side of the equal sign. If we wish the sum of the terms containing a variable to be positive, we use the addition property to eliminate the variable term with the smaller numerical coefficient from one side of the equation. Since \(-3\) is smaller than 2, we added \(3p\) to both sides of the equation. This eliminated \(-3p\) from the right side of the equation and resulted in the sum of the variable terms on the left side of the equation, \(5p\), being positive.

EXAMPLE 4 

Solve the equation \(2(x - 5) + 3 = 3x + 9\).

**Solution**

\[ 2(x - 5) + 3 = 3x + 9 \]

Step 2

\[ 2x - 10 + 3 = 3x + 9 \]

Distributive property was used.

Step 3

\[ 2x - 7 = 3x + 9 \]

Like terms were combined.

Step 4

\[ -7 = x + 9 \]

2x was subtracted from both sides.

Step 4

\[ -16 = x \]

9 was subtracted from both sides.

The solution is \(-16\).

Now Try Exercise 35
Section 2.5 Solving Linear Equations with the Variable on Both Sides of the Equation

EXAMPLE 5 → Solve the equation $7 - 2x + 5x = -2(-3x + 4)$.

Solution

Step 2 $7 - 2x + 5x = 6x - 8$ Distributive property was used.

Step 3 $7 + 3x = 6x - 8$ Like terms were combined.

Step 4 $7 = 3x - 8$ $3x$ was subtracted from both sides.

Step 4 $15 = 3x$ $8$ was added to both sides.

Step 5 $5 = x$ Both sides were divided by $3$.

The solution is $5$.

→ Now Try Exercise 63

2 Solve Equations Containing Decimal Numbers or Fractions

Now we will solve an equation that contains decimal numbers. As explained in the previous section, equations containing decimal numbers may be solved by a number of different procedures. We will illustrate two procedures for solving Example 6.

EXAMPLE 6 → Solve the equation $5.74x + 5.42 = 2.24x - 9.28$.

Solution

Method 1 We first notice that there are no like terms on the same side of the equal sign that can be combined. We will elect to collect the terms containing the variable on the left side of the equation.

$5.74x + 5.42 = 2.24x - 9.28$

Step 4 $5.74x - 2.24x + 5.42 = 2.24x - 2.24x - 9.28$ Subtract 2.24x from both sides.

$3.50x + 5.42 = -9.28$

Step 4 $3.50x + 5.42 = -9.28 - 5.42$ Subtract 5.42 from both sides.

$3.50x = -14.70$

Step 5 $\frac{3.50}{3.50} = \frac{-14.70}{3.50}$ Divide both sides by 3.50.

$x = -4.20$

The solution is $-4.20$.

Method 2 In the previous section, we introduced a procedure to eliminate decimal numbers from equations. If the equation contains decimals given in tenths, multiply both sides of the equation by 10. If the equation contains decimals given in hundredths, multiply both sides of the equation by 100, and so on. Since the given equation has numbers given in hundredths, we will multiply both sides of the equation by 100.

$5.74x + 5.42 = 2.24x - 9.28$

$100(5.74x + 5.42) = 100(2.24x - 9.28)$ Multiply both sides by 100.

$100(5.74x) + 100(5.42) = 100(2.24x) - 100(9.28)$ Distributive property.

$574x + 542 = 224x - 928$

Step 4 $574x + 542 - 542 = 224x - 928 - 542$ Subtract 542 from both sides.

$574x = 224x - 1470$

Step 4 $574x - 224x = 224x - 224x - 1470$ Subtract 224x from both sides.

$350x = -1470$

Step 5 $\frac{350}{350} = \frac{-1470}{350}$ Divide both sides by 350.

$x = -4.20$

Notice we obtain the same answer using either method. You may use either method to solve equations of this type.

→ Now Try Exercise 25
Now let’s solve some equations that contain fractions.

**EXAMPLE 7** Solve the equation \( \frac{1}{2}a = \frac{3}{4}a + \frac{1}{5} \).

**Solution**

Step 1 In this equation we are solving for \( a \). The least common denominator is 20. Begin by multiplying both sides of the equation by the LCD.

\[
\frac{1}{2}a = \frac{3}{4}a + \frac{1}{5}
\]

Step 2

\[
20 \left( \frac{1}{2}a \right) = 20 \left( \frac{3}{4}a + \frac{1}{5} \right)
\]

Multiply both sides by the LCD, 20.

Step 3

\[
10a = 20 \left( \frac{3}{4}a \right) + 20 \left( \frac{1}{5} \right)
\]

Distributive property

\[
10a = 15a + 4
\]

Step 4

\[-5a = 4\]

15a was subtracted from both sides.

Step 5

\[a = \frac{-4}{5}\]

Both sides were divided by \(-5\).

Step 6 Check:

\[
\frac{1}{2}a = \frac{3}{4}a + \frac{1}{5}
\]

\[
\frac{1}{2} \left( \frac{-4}{5} \right) = \frac{3}{4} \left( \frac{-4}{5} \right) + \frac{1}{5}
\]

\[
-\frac{2}{5} = -\frac{3}{5} + \frac{1}{5}
\]

\[-\frac{2}{5} = -\frac{2}{5}\]

True

The solution is \( \frac{-4}{5} \).

** Helpful Hint **

The equation in Example 7, \( \frac{1}{2}a = \frac{3}{4}a + \frac{1}{5} \), could have been written as \( \frac{a}{2} = \frac{3a}{4} + \frac{1}{5} \) because \( \frac{1}{2}a \) is the same as \( \frac{a}{2} \) and \( \frac{3}{4}a \) is the same as \( \frac{3a}{4} \). You would solve the equation \( \frac{a}{2} = \frac{3a}{4} + \frac{1}{5} \) the same way you solved the equation in Example 7.

You would begin by multiplying both sides of the equation by the LCD, 20.

**EXAMPLE 8** Solve the equation \( \frac{x}{4} + 3 = 2(x - 2) \).

**Solution** We will begin by multiplying both sides of the equation by the LCD, 4.

\[
\frac{x}{4} + 3 = 2(x - 2)
\]

\[
4 \left( \frac{x}{4} + 3 \right) = 4 \left[2(x - 2) \right]
\]

Multiply both sides by the LCD, 4.
Section 2.5 Solving Linear Equations with the Variable on Both Sides of the Equation

\[
4\left(\frac{x}{4}\right) + 4(3) = 4(2(x - 2)) \\
x + 12 = 8(x - 2) \\
x + 12 = 8x - 16 \\
12 = 7x - 16 \\
28 = 7x \\
4 = x
\]

A check will show that 4 is the solution.

**Helpful Hint**

Notice the equation in Example 8 had two terms on the left side of the equal sign, \(\frac{x}{4}\) and 3. The equation had only one term on the right side of the equal sign, \(2(x - 2)\). Therefore, after we multiplied both sides of the equation by 4, the next step was to use the distributive property on the left side of the equation.

In Example 8, we began the solution by multiplying both sides of the equation by the LCD. In Example 9, we will solve the same equation, but this time we will begin by using the distributive property.

**Example 9** Solve the equation in Example 8, \(\frac{x}{4} + 3 = 2(x - 2)\), by first using the distributive property.

**Solution** Begin by using the distributive property.

\[
\frac{x}{4} + 3 = 2(x - 2) \\
\frac{x}{4} + 3 = 2x - 4 \\
4\left(\frac{x}{4} + 3\right) = 4(2x - 4) \\
4\left(\frac{x}{4}\right) + 4(3) = 4(2x) - 4(4) \\
x + 12 = 8x - 16 \\
12 = 7x - 16 \\
28 = 7x \\
4 = x
\]

The solution is 4.

Notice that we obtained the same answer in Examples 8 and 9. You may work problems of this type using either procedure unless your instructor asks you to work problems of this type using a specific method.
EXAMPLE 10  Solve the equation $\frac{1}{2}(2x + 3) = \frac{2}{3}(x - 6) + 4$.

Notice that this equation contains one term on the left side of the equal sign and two terms on the right side of the equal sign.

Solution  We will work this problem by first using the distributive property.

\[
\begin{align*}
\frac{1}{2}(2x + 3) &= \frac{2}{3}(x - 6) + 4 \\
\frac{1}{2}(2x) + \frac{1}{2}(3) &= \frac{2}{3}(x) - \frac{2}{3}(6) + 4 \\
\frac{1}{2} \cdot 2x + \frac{1}{2} \cdot 3 &= \frac{2}{3} \cdot x - \frac{2}{3} \cdot 6 + 4 \\
x + \frac{3}{2} &= \frac{2}{3}x - 4 + 4 \\
x + \frac{3}{2} &= \frac{2}{3}x
\end{align*}
\]

Like terms were combined.

\[
\begin{align*}
6 \left(x + \frac{3}{2}\right) &= 6 \left(\frac{2}{3}x\right) \\
6x + 6 \left(\frac{3}{2}\right) &= 6 \left(\frac{2}{3}x\right) \\
6x + 9 &= 4x \\
2x + 9 &= 0 \\
x &= -\frac{9}{2}
\end{align*}
\]

Check: Substitute $-\frac{9}{2}$ for each $x$ in the equation.

\[
\begin{align*}
\frac{1}{2}(2x + 3) &= \frac{2}{3}(x - 6) + 4 \\
\frac{1}{2}(2(-\frac{9}{2}) + 3) &= \frac{2}{3} \left(-\frac{9}{2} - 6\right) + 4 \\
\frac{1}{2}(-9 + 3) &= \frac{2}{3} \left(-\frac{9}{2} - \frac{12}{2}\right) + 4 \\
\frac{1}{2}(-6) &= \frac{2}{3} \left(-\frac{21}{2}\right) + 4 \\
-3 &= -\frac{7}{4} + 4 \\
-3 &= -3
\end{align*}
\]

The solution is $-\frac{9}{2}$.

Now Try Exercise 75

In Example 10, we began by using the distributive property. We could have also begun by multiplying both sides of the equation by the LCD, 6, before using the distributive property. Work Example 10 again now by first multiplying both sides of the equation by the LCD, 6. You should obtain the same answer, $x = -\frac{9}{2}$. 
Section 2.5 Solving Linear Equations With the Variable on Both Sides of the Equation

Example 10 could have also been written as $\frac{2x + 3}{2} = \frac{2(x - 6)}{3} + 4$. If you were given the equation in this form, you could begin by using the distributive property on $\frac{2(x - 6)}{3}$ or you could begin by multiplying both sides of the equation by the LCD, 6. Because this is just another way of writing the equation in Example 10, the answer would be $-\frac{9}{2}$.

We will discuss solving equations containing fractions in more detail later in the book.

3 Identify Identities and Contradictions

Thus far all the equations we have solved have had a single value for a solution. Equations of this type are called conditional equations, for they are only true under specific conditions. Some equations, as in Example 11, are true for infinitely many values of $x$. Equations that are true for infinitely many values of $x$ are called identities. A third type of equation, as in Example 12, has no solution and is called a contradiction.

EXAMPLE 11 Solve the equation $5x - 5 - 2x = 3(x - 2) + 1$.

Solution

\[
\begin{align*}
5x - 5 - 2x &= 3(x - 2) + 1 \\
5x - 5 - 2x &= 3x - 6 + 1 \\
3x - 5 &= 3x - 5 & \text{Distributive property was used.} \\
3x - 5 &= 3x - 5 & \text{Like terms were combined.}
\end{align*}
\]

Since the same expression appears on both sides of the equal sign, the statement is true for infinitely many values of $x$. If we continue to solve this equation further, we might obtain

\[
\begin{align*}
3x - 5 &= 3x - 5 \\
3x &= 3x \\
0 &= 0 & \text{3x was subtracted from both sides.}
\end{align*}
\]

NOTE: The solution process could have been stopped at $3x - 5 = 3x - 5$. Since one side is identical to the other side, the equation is true for infinitely many values of $x$. **The solution to this equation is all real numbers.** When solving an equation like the equation in Example 11, that is always true, write your answer as “all real numbers.”

Now Try Exercise 47

EXAMPLE 12 Solve the equation $-2x + 5 + 3x = 5x - 4x + 7$.

Solution

\[
\begin{align*}
-2x + 5 + 3x &= 5x - 4x + 7 \\
x + 5 &= x + 7 \\
x - x + 5 &= x - x + 7 & \text{Subtract x from both sides.} \\
5 &= 7 & \text{False}
\end{align*}
\]

NOTE: When solving an equation, if you obtain an obviously false statement, as in this example, the equation has no solution. No value of $x$ will make the equation a true statement. **When solving an equation like the equation in Example 12, that is never true, write your answer as “no solution.”** An answer left blank may be marked wrong.

Now Try Exercise 31

Helpful Hint

Some students start solving equations correctly but do not complete the solution. Sometimes they are not sure that what they are doing is correct and they give up for lack of confidence. You must have confidence in yourself. As long as you follow the procedure on page 129, you should obtain the correct solution even if it takes quite a few steps. Remember two important things: (1) your goal is to isolate the variable, and (2) whatever you do to one side of the equation you must also do to the other side. That is, you must treat both sides of the equation equally.
Chapter 2  Solving Linear Equations and Inequalities

EXERCISE SET 2.5

Concept/Writing Exercises

1. a) Write the general procedure for solving an equation that does not contain fractions where the variable appears on both sides of the equation.
   b) Refer to page 129 to see whether you omitted any steps.

2. What is a conditional equation?

3. a) What is an identity?
   b) What is the solution to the equation \(3x + 5 = 3x + 5\)?

4. When solving an equation, how will you know if the equation is an identity?

5. Explain why the equation \(x + 5 = x + 5\) must be an identity.

6. a) What is a contradiction?

b) What is the solution to a contradiction?

7. When solving an equation, how will you know if the equation has no solution?

8. Explain why the equation \(x + 5 = x + 4\) must be a contradiction.

9. a) Explain, step-by-step, how to solve the equation \(4x + 3(x + 2) = 5x - 10\).
   b) Solve the equation by following the steps you listed in part a).

10. a) Explain, step-by-step, how to solve the equation \(4(x + 3) = 6(x - 5)\).
    b) Solve the equation by following the steps you listed in part a).

Practice the Skills

Solve each equation.

11. \(3x = -2x + 15\)

12. \(x + 4 = 2x - 7\)

13. \(-4x + 10 = 6x\)

14. \(3a = 4a + 8\)

15. \(5x + 3 = 6\)

16. \(-6x = 2x + 16\)

17. \(21 - 6p = 3p - 2p\)

18. \(8 - 3x = 4x + 50\)

19. \(2x - 4 = 3x - 6\)

20. \(5x + 7 = 3x + 5\)

21. \(6 - 2y = 9 - 8y + 6y\)

22. \(-4 + 2y = 2y - 6 + y\)

23. \(124.8 - 9.4x = 4.8x + 32.5\)

24. \(9 - 0.5x = 4.5x + 8.5\)

25. \(0.62x - 0.65 = 9.75 - 2.63x\)

26. \(8.71 - 2.44x = 11.02 - 5.74x\)

27. \(5x + 3 = 2(x + 6)\)

28. \(x - 14 = 3(x + 2)\)

29. \(4y - 2 - 8y = 19 + 5y - 3\)

30. \(3x - 5 + 9x = 2 + 4x + 9\)

31. \(2(x - 2) = 4x - 6 - 2x\)

32. \(4r - 10 = 2(r - 4)\)

33. \(-4 + 2y = 2y - 6 + y\)

34. \(7(-3m + 5) = 3(10 - 6m)\)

35. \(-3(2r - 5) + 5 = 3r + 13\)

36. \(4(x - 3) + 2 = 2x + 8\)

37. \(\frac{a}{2} = \frac{a - 3}{2}\)

38. \(\frac{b}{16} = \frac{b - 6}{4}\)

39. \(\frac{n}{10} = \frac{9 - n}{5}\)

40. \(\frac{6 - x}{4} = \frac{x}{8}\)

41. \(\frac{5 - x}{3} = 3x\)

42. \(\frac{y}{4} - 3 = -2x\)

43. \(\frac{5}{8} + \frac{1}{4} = \frac{1}{2}a\)

44. \(\frac{3}{4}x + \frac{1}{2} = \frac{1}{2}x\)

45. \(0.1(x + 10) = 0.3x - 4\)

46. \(5(3.2x - 3) = 2(x - 4)\)

47. \(2(x + 4) = 4x + 3 - 2x + 5\)

48. \(3(y + 1) - 9 = 8y + 6 - 5y\)

49. \(5(3n + 3) = 2(5n - 4) + 6n\)

50. \(-4(-3z - 5) = -(10z + 8) - 2z\)

51. \(-(3 - p) = -(2p + 3)\)

52. \(12 - 2x - 3(x + 2) = 4x + 6 - x\)

53. \(-(x + 4) + 5 = 4x + 1 - 5x\)

54. \(18x + 3(4x - 9) = -6x + 81\)

55. \(35(2x - 1) = 7(x + 4) + 3x\)

56. \(10(x - 10) + 5 = 5(2x - 20)\)

57. \(0.4(x + 0.7) = 0.6(x - 4.2)\)

58. \(0.5(6x - 8) = 1.4(x - 5) - 0.2\)

59. \(\frac{3}{5}x - 2 = x + \frac{1}{3}\)

60. \(\frac{3}{5}x + 4 = \frac{1}{5}x + 5\)

61. \(\frac{1}{5} + 2 = 3(y - 4)\)

62. \(2(x - 4) = \frac{x}{5} + 10\)

63. \(12 - 3x + 7x = -2(-5x + 6)\)

64. \(-2x - 3 - x = -3(-2x + 7)\)

65. \(3(x - 6) - 4(3x + 1) = x - 22\)

66. \(-2(-3x + 5) + 6 = 4(x - 2)\)

67. \(5 + 2x = 6(x + 1) - 5(x - 3)\)

68. \(4 - (6x + 6) = -(2x + 10)\)

69. \(7 - (-y - 5) = 2(y + 3) - 6(y + 1)\)

70. \(12 - 6x + 3(2x + 3) = 2x + 5\)

71. \(\frac{3}{5}(x - 6) = \frac{2}{3}(3x - 5)\)

72. \(\frac{1}{2}(2d + 4) = \frac{1}{3}(4d - 4)\)
Section 2.5 Solving Linear Equations with the Variable on Both Sides of the Equation

73. \[\frac{3(x - 5)}{5} = \frac{3x - 6}{4}\]
74. \[\frac{3(x - 4)}{4} = \frac{5(2x - 3)}{3}\]
75. \[\frac{2}{7}(5x + 4) = \frac{1}{2}(3x - 4) + 1\]

\[\frac{5}{12}(x + 2) = \frac{2}{3}(2x + 1) + \frac{1}{6}\]
76. \[\frac{a - 5}{2} = \frac{3a}{4} + \frac{a - 25}{6}\]
\[x + x + 1 = x + 2.\]
77. \[\frac{a - 7}{3} = \frac{a + 5}{2} - \frac{7a - 1}{6}\]

Problem Solving

79. a) Construct a conditional equation containing three terms on the left side of the equal sign and two terms on the right side of the equal sign.
   b) Explain how you know your answer to part a) is a conditional equation.
   c) Solve the equation.

80. a) Construct a conditional equation containing two terms on the left side of the equal sign and three terms on the right side of the equal sign.
   b) Explain how you know your answer to part a) is a conditional equation.
   c) Solve the equation.

81. a) Construct an identity containing three terms on the left side of the equal sign and two terms on the right side of the equal sign.
   b) Explain how you know your answer to part a) is an identity.
   c) What is the solution to the equation?

82. a) Construct an identity containing two terms on the left side of the equal sign and three terms on the right side of the equal sign.
   b) Explain how you know your answer to part a) is an identity.
   c) What is the solution to the equation?

83. a) Construct a contradiction containing three terms on the left side of the equal sign and two terms on the right side of the equal sign.
   b) Explain how you know your answer to part a) is a contradiction.
   c) What is the solution to the equation?

84. a) Construct a contradiction containing three terms on the left side of the equal sign and four terms on the right side of the equal sign.
   b) Explain how you know your answer to part a) is a contradiction.
   c) What is the solution to the equation?

Challenge Problems

85. Solve the equation \(58 - 1 = 48 + 5\theta\) for \(\theta\).
86. Solve the equation \(2\Delta - 4 = 3\Delta + 5 - \Delta\) for \(\Delta\).
87. Solve the equation \(3\Theta - 5 = 2\Theta - 3 + \Theta\) for \(\Theta\).
88. Solve \(-2(x + 3) + 5x = 3(4 - 2x) - (x + 2)\).
89. Solve \(4 - [5 - 3(x + 2)] = x - 3\).

Group Activity

Discuss and answer Exercise 90 as a group. In the next chapter, we will be discussing procedures for writing application problems as equations. Let’s get some practice now.

90. Chocolate Bars Consider the following word problem. Mary Kay purchased two large chocolate bars. The total cost of the two chocolate bars was equal to the cost of one chocolate bar plus $6. Find the cost of one chocolate bar.
   a) Each group member: Represent this problem as an equation with the variable \(x\).
   b) Each group member: Solve the equation you determined in part a).
   c) As a group, check your equation and your answer to make sure that it makes sense.

Cumulative Review Exercises

[1.5] 91. Evaluate
   a) \(|4|\)
   b) \(|-7|\)
   c) \(|0|\).
[1.9] 92. Evaluate \(\left(\frac{2}{3}\right)^5\) on your calculator.
[2.1] 93. Explain the difference between factors and terms.

94. Simplify \(2(x - 3) + 4x - (4 - x)\).
95. Solve \(2(x - 3) + 4x - (4 - x) = 0\).
96. Solve \((x + 4) - (4x - 3) = 16\).
2.6 Formulas

1. Use the simple interest formula and the distance formula.
2. Use geometric formulas.
3. Solve for a variable in a formula.

A formula is an equation commonly used to express a specific relationship mathematically. For example, the formula for the area of a rectangle is

\[ \text{area} = \text{length} \cdot \text{width} \quad \text{or} \quad A = lw \]

To evaluate a formula, substitute the appropriate numerical values for the variables and perform the indicated operations.

1. Use the Simple Interest Formula and the Distance Formula

A formula commonly used in banking is the simple interest formula.

**Simple Interest Formula**

\[ \text{interest} = \text{principal} \cdot \text{rate} \cdot \text{time} \quad \text{or} \quad i = prt \]

This formula is used to determine the simple interest, \(i\), earned on some savings accounts, or the simple interest an individual must pay on certain loans. In the simple interest formula \(i = prt\), \(p\) is the principal (the amount invested or borrowed), \(r\) is the interest rate in decimal form, and \(t\) is the amount of time of the investment or loan.

**EXAMPLE 1** \(\triangleright\) Auto Loan

To buy a car, Mary Beth Orrange borrowed $10,000 from a bank for 3 years. The bank charged 5% simple annual interest for the loan. How much interest will Mary Beth owe the bank?

**Solution** Understand and Translate Since the bank charged simple interest, we use the simple interest formula to solve the problem. We are given that the rate, \(r\), is 5%, or 0.05 in decimal form. The principal, \(p\), is $10,000 and the time, \(t\), is 3 years. We substitute these values in the simple interest formula and solve for the interest, \(i\).

\[
i = prt
\]

Carry Out

\[
i = 10,000(0.05)(3)
\]

\[
i = 1500
\]

Check There are various ways to check this problem. First ask yourself “Is the answer realistic?” $1500 is a realistic answer. The interest on $10,000 for 1 year at 5% is $500. Therefore for 3 years, an interest of $1500 is correct.

**Answer** Mary Beth will pay $1500 interest. After 3 years, when she repays the loan, she will pay the principal, $10,000, plus the interest, $1500 for a total of $11,500.

**EXAMPLE 2** \(\triangleright\) Savings Account

John Starmack invests $4000 in a savings account that earns simple interest for 2 years. If the interest earned from the account is $500, find the rate.

**Solution** Understand and Translate We use the simple interest formula, \(i = prt\). We are given the principal, \(p\), the time, \(t\), and the interest, \(i\). We are asked to find the rate, \(r\). We substitute the given values in the simple interest formula and solve the resulting equation for \(r\).
Carry Out

\[ i = \text{prt} \]
\[ 500 = 4000(r)(2) \]
\[ 500 = 8000r \]
\[ \frac{500}{8000} = \frac{8000r}{8000} \]
\[ 0.0625 = r \]

**Check and Answer**  The simple interest rate of 0.0625 or 6.25% per year is realistic. If we substitute \( p = 4000, r = 0.0625 \) and \( t = 2 \), we obtain the interest, \( i = 500 \). Thus, the answer checks. The simple interest rate is 6.25%.

We will use the simple interest formula again in Section 3.4.

Another important formula is the distance formula.

**Distance Formula**

distance = rate \( \times \) time \hspace{1cm} \text{or} \hspace{1cm} d = r \times t

Example 3 illustrates the use of the distance formula.

**EXAMPLE 3   Auto Race**  At a NASCAR auto race, Dale Earnhart, Jr., completed the race in 3.2 hours at an average speed of 156.25 miles per hour. Determine the distance of the race.

**Solution**  **Understand and Translate**  We are given the rate, 156.25 miles per hour, and the time is 3.2 hours. We are asked to find the distance.

\[ \text{distance} = \text{rate} \times \text{time} \]
\[ = (156.25)(3.2) = 500 \]

**Answer**  Thus, the distance of the race was 500 miles.

Let’s look at the units in Example 3. The rate is given in miles per hour and the time is given in hours. If we analyze the units (a process called *dimensional analysis*), we see that the answer is given in miles.

\[ \text{distance} = \text{rate} \times \text{time} \]
\[ = \frac{\text{miles}}{\text{hour}} \times \text{hour} \]
\[ = \text{miles} \]

We will use the distance formula again in Section 3.4.

Now we will discuss geometric formulas that will be used throughout the book.

2 **Use Geometric Formulas**

The **perimeter**, \( P \), is the sum of the lengths of the sides of a figure. Perimeters are measured in the same common unit as the sides. For example, perimeter may be measured in centimeters, inches, or feet. The **area**, \( A \), is the total surface within the figure’s boundaries. Areas are measured in square units. For example, area may be measured in square centimeters, square inches, or square feet. **Table 2.1** on page 142 gives the formulas for finding the areas and perimeters of triangles and quadrilaterals. **Quadrilateral** is a general name for a four-sided figure.

In **Table 2.1**, the letter \( h \) is used to represent the **height** of the figure. In the figure of the trapezoid, the sides \( b \) and \( d \) are called the **bases** of the trapezoid. In the triangle, the side labeled \( b \) is called the **base** of the triangle.
EXAMPLE 4 Building an Exercise Area

Dr. Alex Taurke, a veterinarian, decides to fence in a large rectangular area in the yard behind his office for exercising dogs that are boarded overnight. The part of the yard to be fenced in will be 40 feet long and 23 feet wide (see Fig. 2.2).

a) How much fencing is needed?

b) How large, in square feet, will the fenced in area be?

Solution

a) Understand To find the amount of fencing required, we need to find the perimeter of the rectangular area to be fenced in. To find the perimeter, \( P \), substitute 40 for the length, \( l \), and 23 for the width, \( w \), in the perimeter formula, \( P = 2l + 2w \).

\[
P = 2(40) + 2(23) = 80 + 46 = 126
\]

Check and Answer By looking at Figure 2.2, we can see that a perimeter of 126 feet is a reasonable answer. Thus, 126 feet of fencing will be needed to fence in the area for the dogs to exercise.

b) To find the fenced in area, substitute 40 for the length and 23 for the width in the formula for the area of a rectangle. Both the length and width are measured in feet. Since we are multiplying an amount measured in feet by a second amount measured in feet, the answer will be in square feet (or \( \text{ft}^2 \)).

\[
A = lw = 40(23) = 920 \text{ square feet (or 920 ft}^2)\]

Based upon the data given, an area of 920 \( \text{ft}^2 \) is reasonable. The area to be fenced in will be 920 square feet.

Now Try Exercise 97
EXAMPLE 5  •  Panoramic Photo  Heather Hunter enlarges rectangular panoramic photos, like the one shown below. One of her enlarged panoramic photos has a perimeter of 116 inches and a length of 40 inches. Find the width of the photo.

Solution  Understand and Translate  The perimeter, $P$, is 116 inches and the length, $l$, is 40 inches. Substitute these values into the formula for the perimeter of a rectangle and solve for the width, $w$.

\[ P = 2l + 2w \]
\[ 116 = 2(40) + 2w \]

Carry Out

\[ 116 = 80 + 2w \]
\[ 116 - 80 = 80 - 80 + 2w \]
\[ 36 = 2w \]
\[ \frac{36}{2} = \frac{2w}{2} \]
\[ 18 = w \]

Check and Answer  By considering what you know about panoramic photos, and comparing their length and width, you should realize that the dimensions of a length of 40 inches and a width of 18 inches is reasonable. The answer is, the width of the photo is 18 inches.

Now Try Exercise 25

EXAMPLE 6  •  Sailboat  A small sailboat has a triangular sail that has an area of 30 square feet and a base of 5 feet (see Fig. 2.3). Determine the height of the sail.

Solution  Understand and Translate  We use the formula for the area of a triangle given in Table 2.1.

\[ A = \frac{1}{2}bh \]
\[ 30 = \frac{1}{2}(5)h \]

Carry Out

\[ 2 \cdot 30 = 2 \cdot \frac{1}{2}(5)h \]
\[ 60 = 5h \]
\[ \frac{60}{5} = \frac{5h}{5} \]
\[ 12 = h \]

Check and Answer  The height of the triangle is 12 feet. By looking at Figure 2.3, and by your knowledge of sailboat sails, you may realize that a sail 12 feet tall and 5 feet wide at the base is reasonable. Thus, the height of the sail is 12 feet.

Now Try Exercise 99
Chapter 2  Solving Linear Equations and Inequalities

Another figure that we see and use daily is the circle. The **circumference**, \( C \), is the length (or perimeter) of the curve that forms a circle. The **radius**, \( r \), is the line segment from the center of the circle to any point on the circle (Fig. 2.4a). The **diameter** of a circle is a line segment through the center whose endpoints both lie on the circle (Fig. 2.4b). Note that the length of the diameter is twice the length of the radius.

The formulas for both the area and the circumference of a circle are given in Table 2.2.

**TABLE 2.2  Formulas for Circles**

<table>
<thead>
<tr>
<th>Circle</th>
<th>Area</th>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( A = \pi r^2 )</td>
<td>( C = 2\pi r )</td>
</tr>
</tbody>
</table>

The value of \( \pi \), symbolized by the Greek lowercase letter \( \pi \), is an irrational number which cannot be exactly expressed as a decimal number or a numerical fraction. \( \pi \) is approximately 3.14.

**USING YOUR CALCULATOR**

Scientific and graphing calculators have a key for finding the value of \( \pi \). If you press the \( \boxed{\pi} \) key, your calculator may display 3.1415927. This is only an approximation of \( \pi \). If you own a scientific or graphing calculator, use the \( \boxed{\pi} \) key when evaluating expressions containing \( \pi \). If your calculator does not have a \( \boxed{\pi} \) key, use 3.14 to approximate it. When evaluating expressions containing \( \pi \), we will use the \( \boxed{\pi} \) key on a calculator to obtain the answer. The final answer displayed in the text or answer section may therefore be slightly different (and more accurate) than yours if you use 3.14 for \( \pi \).

**EXAMPLE 7  Pizza** A Pizza Hut large pizza has a diameter of 14 inches. Determine the area and circumference of the pizza.

**Solution** The radius is half its diameter, so \( r = \frac{14}{2} = 7 \) inches.

\[
\begin{align*}
A &= \pi r^2 \\
A &= \pi (7)^2 \\
A &= \pi (49) \\
A &\approx 153.94 \text{ square inches}
\end{align*}
\]

\[
\begin{align*}
C &= 2\pi r \\
C &= 2\pi (7) \\
C &= 2\pi (49) \\
C &\approx 43.98 \text{ inches}
\end{align*}
\]

To obtain our answers 153.94 and 43.98, we used the \( \boxed{\pi} \) key on a calculator and rounded our final answer to the nearest hundredth. If you do not have a calculator with a \( \boxed{\pi} \) key and use 3.14 for \( \pi \), your answer for the area would be 153.86.

> Now Try Exercise 101

**EXAMPLE 8  Spaceship Earth** The inside of Spaceship Earth at Epcot Center in Disney World, Florida, is a sphere with a diameter of 165 feet (see photo). Determine the volume of Spaceship Earth.

**Solution** Understand and Translate Table 2.3 gives the formula for the volume of a sphere. The formula involves the radius. Since the diameter is 165 feet, its radius is \( \frac{165}{2} = 82.5 \) feet.
Check and Answer  The volume inside the sphere is very large, 2,352,071.15 cubic feet. Since a rollercoaster-type ride is inside the sphere, the volume must be very large, and so the answer is reasonable.

Now Try Exercise 109

### TABLE 2.3  Formulas for Volumes of Three-Dimensional Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Sketch</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular solid</td>
<td><img src="image" alt="Rectangular solid" /></td>
<td>$V = lwh$</td>
</tr>
<tr>
<td>Right circular cylinder</td>
<td><img src="image" alt="Right circular cylinder" /></td>
<td>$V = \pi r^2h$</td>
</tr>
<tr>
<td>Right circular cone</td>
<td><img src="image" alt="Right circular cone" /></td>
<td>$V = \frac{1}{3}\pi r^2h$</td>
</tr>
<tr>
<td>Sphere</td>
<td><img src="image" alt="Sphere" /></td>
<td>$V = \frac{4}{3}\pi r^3$</td>
</tr>
</tbody>
</table>

3 Solve for a Variable in a Formula

Often in this course and in other mathematics and science courses, you will be given an equation or formula solved for one variable and have to solve it for a different variable. We will now learn how to do this. This material will reinforce what you learned about solving equations earlier in this chapter. We will use the procedures learned here to solve problems in many other sections of the text.

To solve for a variable in a formula, treat each of the quantities, except the one for which you are solving, as if they were constants. Then solve for the desired variable by isolating it on one side of the equation.

**EXAMPLE 9  Distance Formula** Solve the distance formula $d = rt$ for $t$.

**Solution** We must get $t$ all by itself on one side of the equal sign. Since $t$ is multiplied by $r$, we divide both sides of the equation by $r$ to isolate the $t$.

\[
\frac{d}{r} = \frac{rt}{r}
\]

Divide both sides by $r$.

\[
\frac{d}{r} = t
\]

Therefore, $t = \frac{d}{r}$.  Now Try Exercise 45
EXAMPLE 10  Perimeter of Rectangle  
The formula for the perimeter of a rectangle is \( P = 2l + 2w \). Solve this formula for the length, \( l \).

Solution  
We must get \( l \) all by itself on one side of the equation. We begin by removing the \( 2w \) from the right side of the equation to isolate the term containing the \( l \).

\[
\begin{align*}
P &= 2l + 2w \\
P - 2w &= 2l + 2w - 2w \\
P - 2w &= 2l \\
\frac{P - 2w}{2} &= \frac{2l}{2} \\
\frac{P - 2w}{2} &= l \quad \text{or} \quad l = \frac{P}{2} - w \\
\end{align*}
\]

Now Try Exercise 53

Some formulas contain fractions. When a formula contains a fraction, we can eliminate the fraction by multiplying both sides of the equation by the least common denominator, as illustrated in Example 11. We use the multiplication property of equality, as explained in Section 2.3.

EXAMPLE 11  
The formula for the area of a triangle is \( A = \frac{1}{2}bh \). Solve this formula for \( h \).

Solution  
We begin by multiplying both sides of the equation by the LCD, 2, to eliminate the fraction. We then isolate the variable \( h \).

\[
\begin{align*}
A &= \frac{1}{2}bh \\
2 \cdot A &= 2 \cdot \frac{1}{2}bh \\
2A &= bh \\
\frac{2A}{b} &= \frac{bh}{b} \\
\frac{2A}{b} &= h
\end{align*}
\]

Thus, \( h = \frac{2A}{b} \).

Now Try Exercise 51

Write Equations in \( y = mx + b \) Form

When discussing graphing later in this book, we will need to solve many equations for the variable \( y \), and write the equation in the form \( y = mx + b \), where \( m \) and \( b \) represent real numbers. Examples of equations in this form are \( y = 2x + 4 \), \( y = -\frac{1}{2}x - 3 \), and \( y = \frac{4}{5}x + \frac{1}{3} \). The procedure to write equations in \( y = mx + b \) form is illustrated in Examples 12 and 13.

EXAMPLE 12  
Solve the equation \( 6x + 3y = 12 \) for \( y \). Write the answer in \( y = mx + b \) form.

Solution  
Begin by isolating the term containing the variable \( y \).
Section 2.6 Formulas

Subtract 6x from both sides.

Divide both sides by 3.

Write as two fractions.

Now Try Exercise 67

Helpful Hint

Notice that in Example 12, when we obtained \( y = \frac{12 - 6x}{3} \), we had solved the equation for \( y \) since the \( y \) was isolated on one side of the equation. When we wrote the answer as \( y = -2x + 4 \), we wrote the equation in \( y = mx + b \) form.

EXAMPLE 13

Solve the equation \( y = \frac{1}{3} \left( y - \frac{1}{4} (x - 6) \right) \) for \( y \). Write the answer in \( y = mx + b \) form.

Solution

Multiply both sides of the equation by the LCD, 12.

\[
12 \left( y - \frac{1}{3} \right) = 12 \cdot \frac{1}{4} (x - 6)
\]

Multiply both sides by 12.

\[
12y - 4 = 3(x - 6)
\]

Distributive property used on left

\[
12y - 4 = 3x - 18
\]

Distributive property used on right

\[
12y = 3x - 14
\]

Add 4 to both sides

\[
y = \frac{3x - 14}{12}
\]

Divide both sides by 12

\[
y = \frac{3x}{12} - \frac{14}{12}
\]

Write as two fractions

\[
y = \frac{1}{4} x - \frac{7}{6}
\]

Now Try Exercise 79

In Example 13, you may wish to use the distributive property on the right side of the equation before multiplying both sides of the equation by the LCD, 12. Try working the example using this method now to see which procedure you prefer.

EXERCISE SET 2.6

Concept/Writing Exercises

1. What is a formula?
2. What does it mean to evaluate a formula?
3. Write the simple interest formula, then indicate what each letter in the formula represents.
4. What is a quadrilateral?
5. Write the distance formula, then indicate what each letter in the formula represents.
6. What is the relationship between the radius and the diameter of a circle?
Chapter 2  Solving Linear Equations and Inequalities

8. a) What is the perimeter of a figure?
   b) What is the area of a figure?
9. By using any formula for area, explain why area is measured in square units.
10. By using any formula for volume, explain why volume is measured in cubic units.

Practice the Skills

Use the formula to find the value of the variable indicated. Use a calculator to save time and where necessary, round your answer to the nearest hundredth.

11. \( d = rt \) (distance formula); find \( d \) when \( r = 60 \) and \( t = 4 \).
12. \( P = 4s \) (perimeter of a square); find \( P \) when \( s = 6 \).
13. \( A = lw \) (area of a rectangle); find \( A \) when \( l = 12 \) and \( w = 8 \).
14. \( A = \pi r^2 \) (area of a circle); find \( A \) when \( r = 5 \).
15. \( i = pt \) (simple interest formula); find \( i \) when \( p = 2000 \), \( r = 0.06 \), and \( t = 3 \).
16. \( c = 2.54i \) (to change inches to centimeters); find \( c \) when \( i = 12 \).
17. \( P = 2l + 2w \) (perimeter of a rectangle); find \( P \) when \( l = 8 \) and \( w = 5 \).
18. \( f = 1.47m \) (to change speed from mph to ft/sec); find \( f \) when \( m = 60 \).
19. \( A = \pi r^2 \) (area of a circle); find \( A \) when \( r = 5 \).
20. \( A = \frac{m + n}{2} \) (mean of two values); find \( A \) when \( m = 16 \) and \( n = 56 \).
21. \( A = \frac{a + b + c}{3} \) (mean of three values); find \( A \) when \( a = 72 \), \( b = 81 \), and \( c = 93 \).

In Exercises 31–36, use Tables 2.1, 2.2, and 2.3 to find the formula for the area or volume of the figure. Then determine either the area or volume.

31. \[ \text{8 ft} \]
32. \[ \text{4 in.} \]
33. \[ \text{4 ft} \]
34. \[ \text{5 ft} \]
35. \[ \text{4 cm} \]
36. \[ \text{1 m} \]

In Exercises 37 and 38, use the formula \( C = \frac{5}{9}(F - 32) \) to find the Celsius temperature (C) equivalent to the given Fahrenheit temperature (F).

37. \( F = 50^\circ \)
38. \( F = 86^\circ \)

In Exercises 39 and 40, use the formula \( F = \frac{9}{5}C + 32 \) to find the Fahrenheit temperature (F) equivalent to the given Celsius temperature (C).

39. \( C = 25^\circ \)
40. \( C = 10^\circ \)

In Exercises 41–44, find the missing quantity. Use the ideal gas law, \( P = \frac{KT}{V} \), where \( P \) is pressure, \( T \) is temperature, \( V \) is volume, and \( K \) is a constant.

41. \( T = 20, K = 2, V = 1 \)
42. \( P = 80, T = 100, V = 5 \)
43. \( T = 30, P = 3, K = 0.5 \)
44. \( P = 100, K = 2, V = 6 \)
Section 2.6 Formulas

In Exercises 45–66, solve for the indicated variable.

45. \( A = lw, \) for \( w \) 46. \( P = 4s, \) for \( s \) 47. \( d = rt, \) for \( t \)
48. \( C = \pi d, \) for \( d \) 49. \( i = prt, \) for \( t \) 50. \( V = \frac{1}{2}vh, \) for \( l \)
51. \( A = \frac{1}{2}bh, \) for \( b \) 52. \( E = IR, \) for \( I \) 53. \( P = 2l + 2w, \) for \( w \)
54. \( PV = KT, \) for \( T \) 55. \( 3 - 2r = n, \) for \( r \) 56. \( 4m + 5n = 25, \) for \( n \)
57. \( y = mx + b, \) for \( b \) 57. \( y = mx + b, \) for \( x \) 59. \( d = a + b + c, \) for \( b \)
60. \( ax + by = c, \) for \( y \) 61. \( ax + by + c = 0, \) for \( y \) 62. \( V = \pi r^2h, \) for \( b \)
63. \( V = \frac{1}{3}\pi r^2h, \) for \( h \) 64. \( A = \frac{m + 2d}{3}, \) for \( d \) 65. \( A = \frac{m + d}{2}, \) for \( m \)
66. \( L = \frac{c + 2d}{4}, \) for \( d \)

In Exercises 67–82, solve each equation for \( y. \) Write the answer in \( mx + b \) form. See Examples 12 and 13.

67. \( 2x + y = 5 \) 68. \( 6x + 2y = -12 \) 69. \( -3x + 3y = -15 \)
68. \( -2y + 4x = -8 \) 69. \( 4x = 6y - 8 \) 70. \( 15 = 3y - x \)
70. \( 5y = -10 + 3x \) 71. \( -2y = -3x - 18 \) 72. \( -6y = 15 - 3x \)
71. \( -12 = -2x - 3y \) 74. \( -8 = -x - 2y \) 75. \( 4x + 3y = 20 \)
73. \( 9.3 = 3 + \frac{1}{4}(x - 4) \) 76. \( y - 3 = \frac{2}{3}(x + 4) \) 77. \( y - \frac{1}{5} = 2\left(x + \frac{1}{3}\right) \)
72. \( y + 5 = \frac{3}{4}\left(x + \frac{1}{2}\right) \)

Problem Solving

83. When using the distance formula, what happens to the distance if the rate is doubled and the time is halved? Explain.
84. When using the simple interest formula, what happens to the simple interest if both the principal and rate are doubled but the time is halved? Explain.
85. Consider the formula for the area of a square, \( A = s^2. \) If the length of the side of a square, \( s, \) is doubled, what is the change in its area? Explain.
86. Consider the formula for the volume of a cube, \( V = s^3. \) If the length of the side of a cube, \( s, \) is doubled, what is the change in its volume? Explain.
87. Which would have the greater area, a square whose side has a length of \( s \) inches, or a circle whose diameter has a length of \( s \) inches? Explain, using a sketch.
88. Which would have the greater area, a square whose diagonal has a length of \( s \) inches, or a circle whose diameter has a length of \( s \) inches? Explain, using a sketch.

In Exercises 89–92, use the simple interest formula.

89. Auto Loan Thang Tran decided to borrow $6000 from Citibank to help pay for a car. His loan was for 3 years at a simple interest rate of 8%. How much interest will Thang pay?
90. Simple Interest Loan Holly Brosamle lent her brother $4000 for a period of 2 years. At the end of the 2 years, her brother repaid the $4000 plus $640 interest. What simple interest rate did her brother pay?
91. Savings Account Mary Seitz invested a certain amount of money in a savings account paying 3% simple interest per year. When she withdrew her money at the end of 3 years, she received $450 in interest. How much money did Mary place in the savings account?
92. Savings Account Peter Ostroushko put $6000 in a savings account earning 3\(\frac{1}{2}\)% simple interest per year. When he withdrew his money, he received $840 in interest. How long had he left his money in the account?

In Exercises 93–96, use the distance formula.

93. Average Speed On her way from Omaha, Nebraska, to Kansas City, Kansas, Peg Hovde traveled 150 miles in 3 hours. What was her average speed?
94. Walk Lisa Feintech went for a walk where she walked at an average speed of 3.4 miles per hour for 2 hours. How far did she walk?
95. Fastest Car The fastest speed recorded on land was about 763.2 miles per hour by a jet powered car called ThrustSSC. If, during the speed trial, the car traveled for 0.01 hour, how far had the car traveled?
96. Fastest Plane The fastest aircraft is the Lockheed SR-71 Blackbird. If, during the speed trial, the plane covered a distance of 660 miles in 0.3 hours, determine the plane’s average speed.
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Use the formulas given in Tables 2.1, 2.2, and 2.3 to work Exercises 97–110.

97. DVD Player A portable DVD player has a screen with a length of 8 inches and a width (or height) of 6 inches. Determine the area of the screen.

98. Television A plasma television has a screen with a length of 34.9 inches and a width (or height) of 19.6 inches. Determine the perimeter of the screen.

99. Yield Sign A yield traffic sign is triangular with a base of 36 inches and a height of 31 inches. Find the area of the sign.

100. Fencing Milt McGowen has a rectangular lot that measures 100 feet by 60 feet. If Milt wants to fence in his lot, how much fencing will he need?

101. Swimming Pool A circular above-ground swimming pool has a diameter of 24 feet. Determine the circumference of the pool.

102. Living Room Table A round living room table top has a diameter of 3 feet. Find the area of the table top.

103. Kite Below we show a kite. Determine the area of the kite.

104. Suitcase A suitcase measures 26 inches long by 19.5 inches wide and 11 inches deep. Determine the volume of the suitcase.

105. Trapezoidal Sign Canter Martin made a sign to display at a baseball game. The sign was in the shape of a trapezoid. Its bases are 4 feet and 3 feet, and its height is 2 feet. Find the area of the sign, \( \text{ft}^2 \).

106. Jacuzzi The inside of a circular jacuzzi is 8 feet in diameter. If the water inside the jacuzzi is 3 feet deep, determine, in cubic feet, the volume of water in the jacuzzi.

107. Banyan Tree The largest banyan tree in the continental United States is at the Edison House in Fort Myers, Florida. The circumference of the aerial roots of the tree is 390 feet. Find the diameter of the aerial roots to the nearest tenth of a foot.

108. Amphitheater The seats in an amphitheater are inside a trapezoidal area as shown in the figure.

109. Basketball Find the volume of a basketball if its diameter is 9 inches.

110. Oil Drum Roberto Sanchez has an empty oil drum that he uses for storage. The oil drum is 4 feet high and has a diameter of 24 inches. Find the volume of the drum in cubic feet.

111. Body Mass Index A person’s body mass index (BMI) is found by multiplying a person’s weight, \( w \), in pounds by 703, then dividing this product by the square of the person’s height, \( h \), in inches.
   a) Write a formula to find the BMI.
   b) Brandy Belmont is 5 feet 3 inches tall and weighs 135 pounds. Find her BMI.

112. Body Mass Index Refer to Exercise 111. Mario Guzza’s weight is 162 pounds, and he is 5 feet 7 inches tall. Find his BMI.
Challenge Problems

113. Cereal Box  A cereal box is to be made by folding the cardboard along the dashed lines as shown in the figure on the right.

a) Using the formula

\[ \text{volume} = \text{length} \cdot \text{width} \cdot \text{height} \]

write an equation for the volume of the box.

b) Find the volume of the box when \( x = 7 \) cm.

c) Write an equation for the surface area of the box.

d) Find the surface area when \( x = 7 \) cm.

Group Activity

114. Square Face on Cube  Consider the following photo. The front of the figure is a square with a smaller black square painted on the center of the larger square. Suppose the length of one side of the larger square is \( A \), and length of one side of the smaller (the black square) is \( B \). Also the thickness of the block is \( C \).

a) Group member one: Determine an expression for the surface area of the black square.

b) Group member two: Determine an expression for the surface area of the larger square (which includes the smaller square).

c) Group member three: Determine the surface area of the larger square minus the black square (the purple area shown).

d) As a group, write an expression for the volume of the entire solid block.

e) As a group, determine the volume of the entire solid block if its length is 1.5 feet and its width is 0.8 feet.

Cumulative Review Exercises

[1.7] 115. Evaluate \( \frac{4}{15} + \frac{2}{3} \).

116. Evaluate \(-6 + 7 - 4 - 3\).

[1.9] 117. Evaluate \([4(12 + 2^2 - 3)]^2\).

[2.4] 118. Solve the equation \( \frac{r}{2} + 2r = 20 \).
2.7 Ratios and Proportions

1. **Understand Ratios**

A **ratio** is a quotient of two quantities. Ratios provide a way to compare two numbers or quantities. The ratio of the number $a$ to the number $b$ may be written

$$a : b, \quad \frac{a}{b}, \quad a:b, \quad \text{or} \quad \frac{a}{b}$$

where $a$ and $b$ are called the **terms of the ratio**. Notice that the symbol $:$ can be used to indicate a ratio.

**EXAMPLE 1**  
Favorite Movies Several children in sixth grade were asked to name their favorite movie of 2004. The results are indicated on the graph in Figure 2.5.

![Favorite Movies Graph](image)

**Solution** We will use our five-step problem-solving procedure.

a) **Understand and Translate** The ratio we are seeking is

$$\text{Number who selected \emph{The Incredibles}} : \text{Number who selected \emph{The Polar Express}}$$

b) **Carry Out** We substitute the appropriate values into the ratio. This gives

$$70 : 45$$

To write the ratio in **lowest terms**, we simplify by dividing each number in the ratio by 5, the greatest number that divides both terms in the ratio. This gives

$$14 : 9$$

**Check and Answer** Our division is correct. The ratio is 14:9.

b) We use the same procedure as in part a). Fifty children selected \emph{Shrek 2}. There were $50 + 70 + 45 + 30$ or 195 children surveyed. Thus, the ratio is $50:195$, which simplifies to

$$10 : 39$$

> Now Try Exercise 33

The answer in Example 1, part a) could have also been written $\frac{14}{9}$ or 14 to 9. The answer in part b) could have also been written $\frac{10}{39}$ or 10 to 39.
Section 2.7 Ratios and Proportions

EXAMPLE 2 ➤ Cholesterol Level There are two types of cholesterol: low-density lipoprotein, (LDL—considered the harmful type of cholesterol) and high-density lipoprotein (HDL—considered the healthy type of cholesterol). Some doctors recommend that the ratio of low- to high-density cholesterol be less than or equal to 4:1. Mr. Suarez’s cholesterol test showed that his low-density cholesterol measured 167 milligrams per deciliter, and his high-density cholesterol measured 40 milligrams per deciliter. Is Mr. Suarez’s ratio of low- to high-density cholesterol less than or equal to the recommended 4:1 ratio?

Solution Understand We need to determine if Mr. Suarez’s low- to high-density cholesterol is less than or equal to 4:1.

Translate Mr. Suarez’s low- to high-density cholesterol is 167:40. To make the second term equal to 1, we divide both terms in the ratio by the second term, 40.

Carry Out

\[
\frac{167}{40} \div 10 \\
= 4.175
\]

Check and Answer Our division is correct. Therefore, Mr. Suarez’s ratio is not less than or equal to the desired 4:1 ratio. ➤ Now Try Exercise 87

EXAMPLE 3 ➤ Gas–Oil Mixture Some power equipment, such as chainsaws and blowers, use a gas–oil mixture to run the engine. The instructions on a particular chainsaw indicate that 5 gallons of gasoline should be mixed with 40 ounces of special oil to obtain the proper gas–oil mixture. Find the ratio of gasoline to oil in the proper mixture.

Solution Understand To express these quantities in a ratio, both quantities must be in the same units. We can either convert 5 gallons to ounces or 40 ounces to gallons.

Translate Let’s change 5 gallons to ounces. Since there are 128 ounces in 1 gallon, 5 gallons of gas equals 5(128) or 640 ounces. The ratio we are seeking is

ounces of gasoline : ounces of oil

Carry Out

\[
\frac{640}{40} = 16 : 1
\]

Check and Answer Our simplification is correct. The correct ratio of gas to oil for this chainsaw is 16:1. ➤ Now Try Exercise 23

EXAMPLE 4 ➤ Gear Ratio The gear ratio of two gears is defined as

\[
gear \text{ ratio} = \frac{\text{number of teeth on the driving gear}}{\text{number of teeth on the driven gear}}
\]

Find the gear ratio of the gears shown in Figure 2.6.

Solution Understand and Translate To find the gear ratio we need to substitute the appropriate values.

Carry Out

\[
gear \text{ ratio} = \frac{\text{number of teeth on driving gear}}{\text{number of teeth on driven gear}} = \frac{60}{8} = 7.5
\]

Thus, the gear ratio is 7.5:1. Gear ratios are generally given as some quantity to 1. If we divide both terms of the ratio by the second term, we will obtain a ratio of some number to 1. Dividing both 15 and 2 by 2 gives a gear ratio of 7.5:1.

Check and Answer The gear ratio is 7.5:1. This means that as the driving gear goes around once the driven gear goes around 7.5 times. (A typical first gear ratio on a passenger car may be 3.545:1.) ➤ Now Try Exercise 27
2 Solve Proportions Using Cross-Multiplication

A proportion is a special type of equation. It is a statement of equality between two ratios. One way of denoting a proportion is $a:b = c:d$, which is read “$a$ is to $b$ as $c$ is to $d$.” In this text we write proportions as

$$\frac{a}{b} = \frac{c}{d}$$

The $a$ and $d$ are referred to as the extremes, and the $b$ and $c$ are referred to as the means of the proportion. In Sections 2.4 and 2.5, we solved equations containing fractions by multiplying both sides of the equation by the LCD to eliminate fractions. For example, for the proportion

$$\frac{x}{3} = \frac{35}{15}$$

Multiply both sides by the LCD, 15.

$$15 \left( \frac{x}{3} \right) = 15 \left( \frac{35}{15} \right)$$

$5x = 35$

$x = 7$

Another method that can be used to solve proportions is cross-multiplication. This process of cross-multiplication gives the same results as multiplying both sides of the equation by the LCD. However, many students prefer to use cross-multiplication because they do not have to determine the LCD of the fractions, and then multiply both sides of the equation by the LCD.

### Cross-Multiplication

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Note that the product of the extremes is equal to the product of the means.

If any three of the four quantities of a proportion are known, the fourth quantity can easily be found.

**EXAMPLE 5** Solve $\frac{x}{3} = \frac{35}{15}$ for $x$ by cross-multiplying.

**Solution**

$$\frac{x}{3} = \frac{35}{15}$$

Check:

$$\frac{x}{3} = \frac{35}{15}$$

$$x \cdot 15 = 3 \cdot 35$$

$$15x = 105$$

$$x = \frac{105}{15} = 7$$

**Now Try Exercise 37**

Before we introduced cross-multiplication, we solved the proportion $\frac{x}{3} = \frac{35}{15}$ by multiplying both sides of the equation by 15. In Example 5, we solved the same proportion using cross-multiplication. Notice we obtained the same solution, 7, in each case. When you solve an equation using cross-multiplication, you are in effect multiplying both sides of the equation by the product of the two denominators, and then dividing out the common factors. However, this process is not shown.
EXAMPLE 6  Solve \( \frac{-8}{3} = \frac{64}{x} \) for \( x \) by cross-multiplying.

Solution

\[
\begin{align*}
-8 \cdot x &= 3 \cdot 64 \\
-8x &= 192 \\
-8x &= 192 \\
-8 &= -8 \\
x &= -24
\end{align*}
\]

Check:

\[
\begin{align*}
\frac{-8}{3} &= \frac{64}{-24} \\
\frac{-8}{3} &= \frac{8}{-3} \\
\text{True}
\end{align*}
\]

NOW TRY EXERCISE 41

3 Solve Applications

Often, practical problems can be solved using proportions. To solve such problems, use the five-step problem-solving procedure we have been using throughout the book. Below we give that procedure with more specific directions for translating problems into proportions.

To Solve Problems Using Proportions

1. Understand the problem.
2. Translate the problem into mathematical language.
   a) First, represent the unknown quantity by a variable (a letter).
   b) Second, set up the proportion by listing the given ratio on the left side of the equal sign, and the unknown and the other given quantity on the right side of the equal sign. When setting up the right side of the proportion, the same respective quantities should occupy the same respective positions on the left and the right. For example, an acceptable proportion might be
   \[
   \frac{\text{miles}}{\text{hour}} = \frac{\text{miles}}{\text{hour}}
   \]

3. Carry out the mathematical calculations necessary to solve the problem.
   a. Once the proportion is correctly written, drop the units and cross-multiply.
   b. Solve the resulting equation.
4. Check the answer obtained in step 3.
5. Make sure you have answered the question.

Note that the two ratios used in a proportion must have the same units. For example, if one ratio is given in miles/hour and the second ratio is given in feet/hour, one of the ratios must be changed before setting up the proportion.

EXAMPLE 7  Painting  A gallon of paint will cover an area of 575 square feet.

a) How many gallons of paint are needed to cover a house with a surface area of 6525 square feet?

b) If a gallon of paint costs $24.99, what will it cost (before tax) to paint the house?

Solution

a) Understand  The given ratio is 1 gallon per 575 square feet. The unknown quantity is the number of gallons necessary to cover 6525 square feet.

\[
\frac{6 \text{ miles}}{1 \text{ hour}}, \text{ is called a rate. However, few books make the distinction between ratios and rates when discussing proportions.}
\]

\[
\frac{6 \text{ miles}}{1 \text{ hour}}, \text{ is called a rate. However, few books make the distinction between ratios and rates when discussing proportions.}
\]
Chapter 2 Solving Linear Equations and Inequalities

Example 6

Translate
Let \( x \) = number of gallons.

\[
\begin{align*}
\text{Given ratio} & : \quad \frac{1 \text{ gallon}}{575 \text{ square feet}} = \frac{x \text{ gallons}}{6525 \text{ square feet}} \quad \text{Unknown} \\
\text{Given quantity} & : \quad \frac{1}{575} = \frac{x}{6525}
\end{align*}
\]

Note how the amount, in gallons, and the area, in square feet, are given in the same relative positions.

Carry Out
\[
\begin{align*}
\frac{1}{575} &= \frac{x}{6525} \\
1(6525) &= 575x & \text{Cross-multiply.} \\
6525 &= 575x & \text{Solve.} \\
6525 &= x \\
\frac{6525}{575} &= x \\
11.3 &\approx x
\end{align*}
\]

Check
Using a calculator, we determine that both ratios in the proportion, \( \frac{1}{575} \) and \( \frac{11.3}{6525} \), have approximately the same value of 0.00173. Thus, the answer of about 11.3 gallons checks.

Answer
The amount of paint needed to cover an area of 6525 square feet is about 11.3 gallons.

b) Assuming the painter only buys full gallons of paint, he will need to buy 12 gallons in order to paint the house. Since each gallon costs $24.99, the cost (before tax) to paint the house is found by multiplication.

\[12 \times 24.99 = 299.88\]

The cost (before tax) to paint the house is $299.88.

Now Try Exercise 61

Example 8

Charity Luncheon
Each year in Tampa, Florida, the New York Yankees host a charity luncheon, with the proceeds going to support the Tampa Boys and Girls Clubs. At the luncheon, the guests meet and get autographs from members of the team. If a particular player signs, on the average, 33 autographs in 4 minutes, how much time must be allowed for him to sign 350 autographs?

Solution
The unknown quantity is the time needed for the player to sign 350 autographs. We are given that, on the average, he signs 33 autographs in 4 minutes. We will use this given ratio in setting up our proportion.

Translate
We will let \( x \) represent the time to sign 350 autographs.

\[
\begin{align*}
\text{Given ratio} & : \quad \frac{33 \text{ autographs}}{4 \text{ minutes}} = \frac{350 \text{ autographs}}{x \text{ minutes}} \\
\end{align*}
\]

Carry Out
\[
\begin{align*}
\frac{33}{4} &= \frac{350}{x} \\
33x &= 4(350) \\
33x &= 1400 \\
x &= \frac{1400}{33} \approx 42.4
\end{align*}
\]

Check and Answer
Using a calculator, we can determine that both ratios in the proportion, \( \frac{33}{4} \) and \( \frac{350}{42.4} \), have approximately the same value of 8.25. Thus, about 42.4 minutes would be needed for the player to sign the 350 autographs.

Now Try Exercise 55
EXAMPLE 9  \textbf{Drug Dosage}  A doctor asks a nurse to give a patient 250 milligrams of the drug simethicone. The drug is available only in a solution whose concentration is 40 milligrams of simethicone per 0.6 milliliter of solution. How many milliliters of solution should the nurse give the patient?

\textbf{Solution}  \textbf{Understand and Translate}  We can set up the proportion using the medication on hand as the given ratio and the number of milliliters needed to be given as the unknown.

\begin{align*}
\text{Given ratio} & \quad \frac{40 \text{ milligrams}}{0.6 \text{ milliliter}} = \frac{250 \text{ milligrams}}{x \text{ milliliters}} \\
\text{Carry Out} & \quad \frac{40}{0.6} = \frac{250}{x} \quad \text{Cross-multiply.} \\
& \quad 40x = 0.6(250) \quad \text{Solve.} \\
& \quad 40x = 150 \\
& \quad x = \frac{150}{40} = 3.75
\end{align*}

\textbf{Check and Answer}  The nurse should administer 3.75 milliliters of the simethicone solution.

\textbf{Helpful Hint}  When you are setting up a proportion, it does not matter which unit in the given ratio is in the numerator and which is in the denominator as long as the units in the other ratio are in the same relative position. For example,\[ \frac{60 \text{ miles}}{1.5 \text{ hours}} = \frac{x \text{ miles}}{4.2 \text{ hours}} \quad \text{and} \quad \frac{1.5 \text{ hours}}{60 \text{ miles}} = \frac{4.2 \text{ hours}}{x \text{ miles}} \]

will both give the same answer of 168 (try it and see). When setting up the proportion, set it up so that it makes the most sense to you. Notice that when setting up a proportion containing different units, the same units should not be multiplied by themselves during cross multiplication.

\textbf{4} \textbf{Use Proportions to Change Units}  Proportions can also be used to convert from one quantity to another. For example, you can use a proportion to convert a measurement in feet to a measurement in meters, or to convert from pounds to kilograms. The following examples illustrate converting units.

\textbf{EXAMPLE 10}  \textbf{Kilometers to Miles}  There are approximately 1.6 kilometers in 1 mile. What is the distance, in miles, of 78 kilometers?

\textbf{Solution}  \textbf{Understand and Translate}  We know that 1 mile $\approx$ 1.6 kilometers. We use this known fact in one ratio of our proportion. In the second ratio, we set the quantities with the same units in the same respective positions. The unknown quantity is the number of miles, which we will call $x$.

\begin{align*}
\text{Known ratio} & \quad \frac{1 \text{ mile}}{1.6 \text{ kilometers}} = \frac{x \text{ miles}}{78 \text{ kilometers}}
\end{align*}

Note that both numerators contain the same units, and both denominators contain the same units.
Chapter 2  Solving Linear Equations and Inequalities

Carry Out  Now drop the units and solve for \( x \) by cross-multiplying.

\[
\frac{1}{1.6} = \frac{x}{78}
\]

Cross-multiply.
\[78 = 1.6x\]
Solve.
\[\frac{78}{1.6} = \frac{1.6x}{1.6}\]
\[48.75 = x\]

Check and Answer  Thus, 78 kilometers equals about 48.75 miles.

EXAMPLE 11  Exchanging Currency  When people travel to a foreign country they often need to exchange currency. Donna Boccio visited Cancun, Mexico. She stopped by a local bank and was told that 1 U.S. could be exchanged for 10.96 pesos.

a) How many pesos would she get if she exchanged $150 U.S.?

b) Later that same day, Donna went to the city market where she purchased a ceramic figurine. The price she negotiated for the figurine was 245 pesos. Using the exchange rate given, determine the cost of the figurine in U.S. dollars.

Solution  

a) Understand  We are told that 1 U.S. can be exchanged for 10.96 Mexican pesos.

We use this known fact for one ratio in our proportion. In the second ratio, we set the quantities with same units in the same respective positions.

Translate  The unknown quantity is the number of pesos, which we shall call \( x \).

\[
\frac{1 \text{ U.S.}}{10.96 \text{ pesos}} = \frac{150 \text{ U.S.}}{x \text{ pesos}}
\]

Note that both numerators contain U.S. dollars and both denominators contain pesos.

Carry Out
\[
\frac{1}{10.96} = \frac{150}{x}
\]
\[1x = 10.96(150)\]
\[x = 1644\]

Check and Answer  Thus, $150 U.S. could be exchanged for 1644 Mexican pesos.

b) Understand and Translate  We use the same given ratio that we used in part \( \text{a)}\). Now we must find the equivalent in U.S. dollars of 245 Mexican pesos. Let's call the equivalent U.S. dollars \( x \).

\[
\frac{1 \text{ U.S.}}{10.96 \text{ pesos}} = \frac{x \text{ U.S.}}{245 \text{ pesos}}
\]

Carry Out
\[
\frac{1}{10.96} = \frac{x}{245}
\]
\[1(245) = 10.96x\]
\[245 = 10.96x\]
\[22.35 \approx x\]

Check and Answer  The cost of the figurine in U.S. dollars is $22.35.
Section 2.7 Ratios and Proportions

Helpful Hint
Some of the problems we have just worked using proportions could have been done without using proportions. However, when working problems of this type, students often have difficulty in deciding whether to multiply or divide to obtain the correct answer. By setting up a proportion, you may be better able to understand the problem and have more success in obtaining the correct answer.

5 Use Proportions to Solve Problems Involving Similar Figures

Proportions can also be used to solve problems in geometry and trigonometry. The following examples illustrate how proportions may be used to solve problems involving similar figures. Two figures are said to be similar when their corresponding angles are equal and their corresponding sides are in proportion. Two similar figures will have the same shape.

EXAMPLE 12 The figures to the left are similar. Find the length of the side indicated by the x.

Solution We set up a proportion of corresponding sides to find the length of side x.

\[
\frac{5 \text{ inches}}{2 \text{ inches}} = \frac{12 \text{ inches}}{x}
\]

\[5x = 24\]
\[x = \frac{24}{5} = 4.8\]

Thus, the side indicated by x is 4.8 inches in length.

Now Try Exercise 51

Note in Example 12 that the proportion could have also been set up as

\[
\frac{5}{12} = \frac{2}{x}
\]

because one pair of corresponding sides is in the numerators and another pair is in the denominators.

EXAMPLE 13 Triangles ABC and AB'C' are similar triangles. Find the length of side AB'.

Solution We set up a proportion of corresponding sides to find the length of side AB'. We will let x represent the length of side AB'. One proportion we can use is

\[
\frac{\text{length of } AB}{\text{length of } BC} = \frac{\text{length of } AB'}{\text{length of } B'C'}
\]

Now we insert the proper values and solve for the variable, x.

\[
\frac{15}{9} = \frac{x}{7.2}
\]
\[(15)(7.2) = 9x
\]
\[108 = 9x
\]
\[12 = x
\]

Thus, the length of side AB' is 12 inches.

Now Try Exercise 53
Chapter 2  Solving Linear Equations and Inequalities

EXERCISE SET 2.7

Concept/Writing Exercises

1. What is a ratio?
2. In the ratio \( a:b \), what are the \( a \) and \( b \) called?
3. List three ways to write the ratio of \( c \) to \( d \).
4. What is a proportion?
5. As you have learned, proportions can be used to solve a wide variety of problems. What information is needed for a problem to be set up and solved using a proportion?
6. What are similar figures?
7. Must similar figures have the same shape? Explain.
8. Must similar figures be the same size? Explain.

In Exercises 9–12, is the proportion set up correctly? Explain.

9. \( \frac{gal}{min} = \frac{gal}{min} \)
10. \( \frac{mi}{hr} = \frac{mi}{hr} \)
11. \( \frac{ft}{sec} = \frac{sec}{ft} \)
12. \( \frac{tax}{cost} = \frac{cost}{tax} \)

Practice the Skills

The results of a mathematics examination are 6 A’s, 4 B’s, 9 C’s, 3 D’s, and 2 F’s. Write the following ratios in lowest terms.

13. A’s to C’s
14. F’s to total grades
15. D’s to A’s
16. Grades better than C to total grades
17. Total grades to D’s
18. Grades better than C to grades less than C

Determine the following ratios. Write each ratio in lowest terms.

19. 7 gallons to 4 gallons
20. 50 dollars to 60 dollars
21. 5 ounces to 15 ounces
22. 18 liters to 24 liters
23. 3 hours to 30 minutes
24. 6 feet to 4 yards
25. 7 dimes to 12 nickels
26. 26 ounces to 4 pounds

In Exercises 27 and 28, find the gear ratio. Write the ratio as some quantity to 1. (See Example 4.)

27. Driving gear, 40 teeth; driven gear, 5 teeth
28. Driving gear, 30 teeth; driven gear, 8 teeth

In Exercises 29–32, a) Determine the indicated ratio, and b) write the ratio as some quantity to 1.

29. American Consumers According to a report issued by the U.S. Department of Agriculture’s Economic Research Service, each year the average American consumer drinks approximately 50 gallons of soft drinks compared to 26 gallons of coffee, 23 gallons of milk, and less than 10 gallons of fruit juices. What is the ratio of the number of gallons of soft drinks consumed to the number of gallons of milk consumed?

30. Mail Letter In January 2006, the cost to mail a one-ounce letter was 39 cents and the cost to mail a two-ounce letter was 63 cents. What is the ratio of the cost to mail a one-ounce letter to the cost to mail a two-ounce letter?

31. Minimum Wage The United States minimum wage in 1985 was $3.35 per hour, and the United States minimum wage in 2005 was $5.15 per hour. What is the ratio of the U.S. minimum wage in 2005 to the U.S. minimum wage in 1985?

32. Population The United States population in 1990 was about 249 million, and the population in 2006 was about 299 million. What is the ratio of the U.S. population in 2006 to the U.S. population in 1990?

33. Traveling the Toll Roads

a) Determine the ratio of the toll rate on the Delaware Turnpike to the toll rate on the Garden State Parkway.
b) Determine the ratio of the toll rate on the Massachusetts Turnpike (Boston ext.) to the toll rate on the New York State Thruway (Current).

How Toll Rates Compare

<table>
<thead>
<tr>
<th>Toll Road</th>
<th>Cents per mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delaware Turnpike</td>
<td>14.2</td>
</tr>
<tr>
<td>Dulles Greenway, VA</td>
<td>16.7</td>
</tr>
<tr>
<td>Pocahontas Parkway, VA</td>
<td>16.7</td>
</tr>
<tr>
<td>MA Turnpike (Boston ext.)</td>
<td>12.5</td>
</tr>
<tr>
<td>Chesapeake Expressway, VA</td>
<td>6.7</td>
</tr>
<tr>
<td>Dulles Toll Road, VA</td>
<td>6.4</td>
</tr>
<tr>
<td>New Jersey Turnpike</td>
<td>5.7</td>
</tr>
<tr>
<td>JFK Highway, MD</td>
<td>4.8</td>
</tr>
<tr>
<td>N.Y. State Thruway (Proposed)</td>
<td>3.9</td>
</tr>
<tr>
<td>N.Y. State Thruway (Current)</td>
<td>3.1</td>
</tr>
<tr>
<td>MA Turnpike (N.Y. state line to Boston)</td>
<td>2.8</td>
</tr>
<tr>
<td>Garden State Parkway</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Source: Rochester Democrat & Chronicle, April, 2005
Section 2.7 Ratios and Proportions

34. Number of Passports Processed
   a) Estimate the ratio of passports processed in the United States in 1996 to those processed in the United States in 2004.
   b) Estimate the ratio of passports processed in the United States in 2002 to those processed in the United States in 1997.

35. Favorite Doughnut
   See figure at the top of the next column.
   a) Determine the ratio of people whose favorite doughnut is glazed to people whose favorite doughnut is filled.
   b) Determine the ratio of people whose favorite doughnut is frosted to people whose favorite doughnut is plain.

Solve each proportion for the variable by cross-multiplying.

37. \( \frac{x}{3} = \frac{20}{5} \)
38. \( \frac{x}{8} = \frac{24}{48} \)
39. \( \frac{3}{x} = \frac{75}{a} \)
40. \( \frac{x}{3} = \frac{90}{30} \)

41. \( \frac{-7}{3} = \frac{21}{p} \)
42. \( \frac{-12}{36} = \frac{36}{x} \)
43. \( \frac{15}{45} = \frac{x}{x} \)
44. \( \frac{x}{6} = \frac{7}{42} \)

45. \( \frac{3}{z} = \frac{-1.5}{27} \)
46. \( \frac{3}{12} = \frac{-1.4}{z} \)
47. \( \frac{9}{12} = \frac{x}{8} \)
48. \( \frac{2}{20} = \frac{x}{200} \)

The following figures are similar. For each pair, find the length of the side indicated by \( x \).

49.

50.

51.

52.
Chapter 2  Solving Linear Equations and Inequalities

53. Problem Solving

In Exercises 55–74, write a proportion that can be used to solve the problem. Then solve the equation to obtain the answer.

55. Washing Clothes  A bottle of liquid Tide contains 100 fluid ounces. If one wash load requires 4 ounces of the detergent, how many washes can be done with one bottle of Tide?

56. Laying Cable  A telephone cable crew is laying cable at a rate of 42 feet an hour. How long will it take them to lay 252 feet of cable?

57. Truck Mileage  A 2004 Chevy S-10 pickup truck with a 4.3-liter engine is rated to get 19 miles per gallon (highway driving). How far can it travel on 14.2 gallons of gas?

58. Purchasing Stock  If 2 shares of stock can be purchased for $38.25, how many shares can be purchased for $344.25?

59. Model Train  A model train set is in a ratio of 1:20. That is, one foot of the model represents 20 feet of the original train. If a caboose is 30 feet long, how long should the model be?

60. Property Tax  The property tax in the city of Hendersonville, North Carolina, is $9.475 per $1000 of assessed value. If the Estever’s house is assessed at $145,000, how much property tax will they owe?

61. Insecticide Application  The instructions on a bottle of liquid insecticide say “use 3 teaspoons of insecticide per gallon of water.” If your sprayer has an 8-gallon capacity, how much insecticide should be used to fill the sprayer?

62. Spreading Fertilizer  If a 40-pound bag of fertilizer covers 5000 square feet, how many pounds of fertilizer are needed to cover an area of 26,000 square feet?

63. Blue Heron  The photograph shows a blue heron. If the blue heron, that measures 3.5 inches in the photo is actually 3.75 feet tall, approximately how long is its beak if it measures 0.4 inch in the photo?

64. Maps  On a map, 0.5 inch represents 22 miles. What will be the length on a map that corresponds to a distance of 55 miles?

65. Onion Soup  A recipe for 6 servings of French onion soup requires 1 1/4 cups of thinly sliced onions. If the recipe were to be made for 15 servings, how many cups of onions would be needed?

66. John Grisham Novel  Karen Estes is currently reading a John Grisham novel. If she reads 72 pages in 1.3 hours, how long will it take her to read the entire 666-page novel?

67. Wall Street Bull  Suppose the famous bull by the New York Stock Exchange (see photo below) is a replica of a real bull in a ratio of 2.95 to 1. That is, the metal bull is 2.95 times greater than the regular bull. If the length of the Wall Street bull is 28 feet long, approximately how long is the bull that served as its model?

68. Flood  When they returned home from vacation, the Dun cans had a foot of water in their basement. They contacted their fire department, which sent equipment to pump out the water. After the pump had been on for 30 minutes, 3 inches of water had been removed. How long, from the time they
started pumping, will it take to remove all the water from the basement?

69. **Drug Dosage**  A nurse must administer 220 micrograms of atropine sulfate. The drug is available in solution form. The concentration of the atropine sulfate solution is 400 micrograms per milliliter. How many milliliters should be given?

70. **Dosage by Body Surface**  A doctor asks a nurse to administer 0.7 gram of meprobamate per square meter of body surface. The patient’s body surface is 0.6 square meter. How much meprobamate should be given?

71. **Reading a Novel**  Mary read 40 pages of a novel in 30 minutes. If she continues reading at the same rate, how long will it take her to read the entire 760-page book?

72. **Swimming Laps**  Jason Abbott swims 3 laps in 2.3 minutes. Approximately how long will it take him to swim 30 laps if he continues to swim at the same rate?

73. **Prader-Willi Syndrome**  It is estimated that each year in the United States about 1 in every 12,000 (1:12,000) people is born with a genetic disorder called Prader-Willi syndrome. If there were approximately 4,063,000 births in the United States in 2005, approximately how many children were born with Prader-Willi syndrome?

74. **Scrapbooking**  Penelope Penna completed 4 scrapbook pages in 20.5 minutes. Approximately how long will it take her to complete 36 scrapbook pages if she continues to complete the scrapbook at the same rate?

In Exercises 75–86, use a proportion to make the conversion. Round your answers to two decimal places.

75. Convert 78 inches to feet.

76. Convert 22,704 feet to miles (5280 feet = 1 mile).

77. Convert 26.1 square feet to square yards (9 square feet = 1 square yard).

78. Convert 146.4 ounces to pounds.

79. **Newborn**  One inch equals 2.54 centimeters. Find the length of a newborn, in inches, if it measures 50.8 centimeters.

80. **Distance**  One mile equals approximately 1.6 kilometers. Find the distance, in kilometers, from San Diego, California, to San Francisco, California—a distance of 520 miles.

San Francisco, California

81. **Home Run Record**  Barry Bonds, who plays for the San Francisco Giant’s baseball team, holds the record for the most home runs, 73, in a 162 game season. In the first 50 games of a season, how many home runs would a player need to hit to be on schedule to break Bonds’ record?

82. **Topsoil**  A 40 pound bag of topsoil covers 12 square feet (one inch deep). How many pounds of the top soil are needed to cover 350 square feet (one inch deep)?

83. **Interest on Savings**  Jim Chao invests a certain amount of money in a savings account. If he earned $110.52 in 180 days, how much interest would he earn in 500 days assuming the interest rate stays the same?

84. **Gold**  If gold is selling for $408 per 480 grains (a troy ounce), what is the cost per grain?

85. **Mexican Pesos**  Suppose that the exchange rate from U.S. dollars to Mexican pesos is $1 per 10.567 pesos. How many pesos would Elizabeth Averbeck receive if she exchanged $200 U.S.?

86. **Currency Exchange**  When Mike Weatherbee visited the United States from Canada, he exchanged $13.50 Canadian for $10 U.S. If he exchanges his remaining $600 Canadian for U.S. dollars, how much more in dollars will he receive?

The Peace Bridge Connecting the United States and Canada

87. **Cholesterol**  Mrs. Ruff’s low-density cholesterol level is 127 milligrams per deciliter (mg/dL). Her high-density cholesterol level is 60 mg/dL. Is Mrs. Ruff’s ratio of low-to-high-density cholesterol level less than or equal to the 4:1 recommended level? (See Example 2.)

88. **Cholesterol**

a) Another ratio used by some doctors when measuring cholesterol level is the ratio of total cholesterol to high-density cholesterol.* Is this ratio increased or decreased if the total cholesterol remains the same but the high-density level is increased? Explain.

b) Doctors recommend that the ratio of total cholesterol to high-density cholesterol be less than or equal to 4.5:1. If Mike’s total cholesterol is 220 mg/dL and his high-density cholesterol is 50 mg/dL, is his ratio less than or equal to 4.5:1? Explain.

89. For the proportion \( \frac{a}{b} = \frac{c}{d} \), if a increases while b and d stay the same, what must happen to c? Explain.

90. For the proportion \( \frac{a}{b} = \frac{c}{d} \), if a and c remain the same while d decreases, what must happen to b? Explain.

*Total cholesterol includes both low- and high-density cholesterol, plus other types of cholesterol.
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Challenge Problems

91. Wear on Tires A new Goodyear tire has a tread of about 0.34 inch. After 5000 miles the tread is about 0.31 inch. If the legal minimum amount of tread for a tire is 0.06 inch, how many more miles will the tires last? (Assume no problems with the car or tires and that the tires wear at an even rate.)

92. Apple Pie The recipe for the filling for an apple pie calls for

1 cup flour   1 1/2 cups sugar
1/2 teaspoon salt
1/4 teaspoon nutmeg  2 tablespoons butter or margarine
1 teaspoon cinnamon

Determine the amount of each of the other ingredients that should be used if only 8 cups of apples are available.

93. Insulin Insulin comes in 10-cubic-centimeter (cc) vials labeled in the number of units of insulin per cubic centimeter. Thus, a vial labeled U40 means there are 40 units of insulin per cubic centimeter of fluid. If a patient needs 25 units of insulin, how many cubic centimeters of fluid should be drawn up into a syringe from the U40 vial?

c) Compare these results with other members of your group.

d) What one ratio would you use to report the height to arm span for your group as a whole? Explain.

94. a) Each group member: Find the ratio of your height to your arm span (finger tips to finger tips) when your arms are extended horizontally outward. You will need help from your group in getting these measurements.

b) If a box were to be drawn about your body with your arms extended, would the box be a square or a rectangle? If a rectangle, would the longer length be your arm span or your height measurement? Explain.

c) Compare these results with other members of your group.

d) What one ratio would you use to report the height to arm span for your group as a whole? Explain.

95. A special ratio in mathematics is called the golden ratio. Do research in a history of mathematics book or on the Internet, and as a group write a paper that explains what the golden ratio is and why it is important.

Cumulative Review Exercises

[1.10] Name each illustrated property.

96. $x + 3 = 3 + x$

97. $(3x) = (3x)y$

98. $2(x - 3) = 2x - 6$

99. Solve $3(4x - 3) = 6(2x + 1) - 15$

100. Solve $y = mx + b$ for $m$.

2.8 Inequalities in One Variable

1. Solve Linear Inequalities

The is-greater-than symbol, $>$, and is-less-than symbol, $<$, were introduced in Section 1.5. The symbol $\geq$ means is greater than or equal to and $\leq$ means is less than or equal to. A mathematical statement containing one or more of these symbols is called an inequality. The direction of the symbol is sometimes called the sense or order of the inequality.

Examples of Inequalities in One Variable

$x + 3 < 5 \quad x + 4 \geq 2x - 6 \quad 4 > -x + 3$

To solve an inequality, we must get the variable by itself on one side of the inequality symbol. To do this, we make use of properties very similar to those used to solve equations. Here are four properties used to solve inequalities. Later in this section, we will introduce two additional properties.

Properties Used to Solve Inequalities

For real numbers, $a$, $b$, and $c$:

1. If $a > b$, then $a + c > b + c$.
2. If $a > b$, then $a - c > b - c$.
3. If $a > b$ and $c > 0$, then $ac > bc$.
4. If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$.
Section 2.8 Inequalities in One Variable

Property 1 says the same number may be added to both sides of an inequality. Property 2 says the same number may be subtracted from both sides of an inequality. Property 3 says the same positive number may be used to multiply both sides of an inequality. Property 4 says the same positive number may be used to divide both sides of an inequality. When any of these four properties is used, the direction of the inequality symbol does not change.

EXAMPLE 1 Solve the inequality \( x - 5 > -2 \), and graph the solution on a number line.

Solution To solve this inequality, we need to isolate the variable, \( x \). Therefore, we must eliminate the \(-5\) from the left side of the inequality. To do this, we add 5 to both sides of the inequality.

\[
\begin{align*}
x - 5 &> -2 \\
x - 5 + 5 &> -2 + 5 \\
x &> 3
\end{align*}
\]

The solution is all real numbers greater than 3. We can illustrate the solution on a number line by placing an open circle at 3 on a number line and drawing an arrow to the right (Fig. 2.7).

The open circle at the 3 indicates that the 3 is not part of the solution. The arrow going to the right indicates that all the values greater than 3 are solutions to the inequality.

Now Try Exercise 13

EXAMPLE 2 Solve the inequality \( 2x + 6 \leq -2 \), and graph the solution on a number line.

Solution To isolate the variable, we must eliminate the \(+6\) from the left side of the inequality. We do this by subtracting 6 from both sides of the inequality.

\[
\begin{align*}
2x + 6 &\leq -2 \\
2x + 6 - 6 &\leq -2 - 6 \\
2x &\leq -8 \\
\frac{2x}{2} &\leq \frac{-8}{2} \\
x &\leq -4
\end{align*}
\]

The solution is all real numbers less than or equal to \(-4\). We can illustrate the solution on a number line by placing a closed, or darkened, circle at \(-4\) and drawing an arrow to the left (Fig. 2.8).

The darkened circle at \(-4\) indicates that \(-4\) is a part of the solution. The arrow going to the left indicates that all the values less than \(-4\) are also solutions to the inequality.

Now Try Exercise 21

Notice in properties 3 and 4 that we specified that \( c > 0 \). What happens when an inequality is multiplied or divided by a negative number? Examples 3 and 4 illustrate that when an inequality is multiplied or divided by a negative number, the direction of the inequality symbol changes.

EXAMPLE 3

a) Multiply both sides of the inequality \( 8 > -4 \) by \(-2\).

b) Divide both sides of the inequality \( 8 > -4 \) by \(-2\).

Solution

a) \[
\begin{align*}
8 &> -4 \\
-2(8) &< -2(-4) \quad \text{Change the direction of the inequality symbol.} \\
-16 &< 8
\end{align*}
\]

b)
b) \[ 8 > -4 \]
\[ \frac{8}{-2} < \frac{-4}{-2} \]
Change the direction of the inequality symbol.
\[ -4 < 2 \]

Now we state two additional properties, used when an inequality is multiplied or divided by a negative number.

**Additional Properties Used to Solve Inequalities**

5. If \( a > b \) and \( c < 0 \), then \( ac < bc \).
6. If \( a > b \) and \( c < 0 \), then \( \frac{a}{c} < \frac{b}{c} \).

**EXAMPLE 4** Solve the inequality \(-2x < 8\), and graph the solution on a number line.

**Solution** To isolate the variable, we must eliminate the \(-2\) on the left side of the inequality. To do this, we can divide both sides of the inequality by \(-2\). When we do this, however, we must remember to change the direction of the inequality symbol.

\[ -2x < 8 \]
\[ \frac{-2x}{-2} > \frac{8}{-2} \]  
\[ x > -4 \]

The solution is all real numbers greater than \(-4\). The solution is graphed on a number line in Figure 2.9.

**EXAMPLE 5** Solve the inequality \(4 \geq -5 - x\), and graph the solution on a number line. We will illustrate two methods that can be used to solve this inequality.

**Solution**

**Method 1:**
\[ 4 \geq -5 - x \]
\[ 4 + 5 \geq -5 + 5 - x \]  
Add 5 to both sides.
\[ 9 \geq -x \]
\[ -1(9) \leq -1(-x) \]  
Multiply both sides by \(-1\) and change the direction of the inequality symbol.
\[ -9 \leq x \]

The inequality \(-9 \leq x\) can also be written \(x \geq -9\).

**Method 2:**
\[ 4 \geq -5 - x \]
\[ 4 + x \geq -5 - x + x \]  
Add \(x\) to both sides.
\[ 4 + x \geq -5 \]
\[ 4 - 4 + x \geq -5 - 4 \]  
Subtract 4 from both sides.
\[ x \geq -9 \]

The solution is graphed on a number line in Figure 2.10. Other methods could also be used to solve this problem.
Section 2.8 Inequalities in One Variable

Notice in Example 5, Method 1, we wrote \(-9 \leq x\) as \(x \geq -9\). Although the solution \(-9 \leq x\) is correct, it is customary to write the solution to an inequality with the variable on the left. One reason we write the variable on the left is that it often makes it easier to graph the solution on the number line. How would you graph \(-3 > x\)? How would you graph \(-5 \leq x\)? If you rewrite these inequalities with the variable on the left side, the answer becomes clearer.

\[
-3 > x \quad \text{means} \quad x < -3 \\
-5 \leq x \quad \text{means} \quad x \geq -5
\]

Notice that you can change an answer from an is-greater-than statement to an is-less-than statement or from an is-less-than statement to an is-greater-than statement. When you change the answer from one form to the other, remember that the inequality symbol must point to the letter or number to which it was pointing originally.

**Helpful Hint**

\[
\begin{align*}
a > x \quad \text{means} \quad x < a & \quad \text{Note that both inequality symbols point to} \ x. \\
a < x \quad \text{means} \quad x > a & \quad \text{Note that both inequality symbols point to} \ a.
\end{align*}
\]

Now let’s solve inequalities where the variable appears on both sides of the inequality symbol. To solve these inequalities, we use the same basic procedure that we used to solve equations. However, we must remember that whenever we multiply or divide both sides of an inequality by a negative number, we must change the direction of the inequality symbol.

**EXAMPLE 6** Solve the inequality \(-5p + 9 < -2p + 6\), and graph the solution on a number line.

**Solution** This inequality uses the variable, \(p\). The variable used does not affect the procedure for solving the inequality.

\[
\begin{align*}
-5p + 9 & < -2p + 6 \\
-5p + 5p + 9 & < -2p + 5p + 6 \\
9 & < 3p + 6 \\
9 - 6 & < 3p + 6 - 6 \\
3 & < 3p \\
\frac{3}{3} & < \frac{3p}{3} \\
1 & < p \\
\text{or} \quad & p > 1
\end{align*}
\]

The solution is graphed in Figure 2.11.
EXAMPLE 7  Solve the inequality \( \frac{1}{2}x + 3 \leq -\frac{1}{3}x + 7 \), and graph the solution on a number line.

Solution  Since the inequality contains fractions, we begin by multiplying both sides of the inequality by the LCD, 6, to eliminate the fractions.

\[
\frac{1}{2}x + 3 \leq -\frac{1}{3}x + 7
\]

Multiply both sides by the LCD, 6.

\[
6\left(\frac{1}{2}x + 3\right) \leq 6\left(-\frac{1}{3}x + 7\right)
\]

Distributive property

\[
3x + 18 \leq -2x + 42
\]

2x was added to both sides.

\[
5x + 18 \leq 42
\]

18 was subtracted from both sides.

\[
x \leq 24
\]

Both sides were divided by 5.

The solution is graphed in Figure 2.12.

Now Try Exercise 53

2 Solve Linear Inequalities That Have All Real Numbers as Their Solution, or Have No Solution

In Examples 8 and 9, we illustrate two special types of inequalities. Example 8 is an inequality that is true for all real numbers, and Example 9 is an inequality that is never true for any real number.

EXAMPLE 8  Solve the inequality \( 2(x + 3) \leq 5x - 3x + 8 \), and graph the solution on a number line.

Solution

\[
2(x + 3) \leq 5x - 3x + 8
\]

Distributive property was used.

\[
2x + 6 \leq 2x + 8
\]

Like terms were combined.

\[
2x + 6 \leq 2x + 8
\]

Subtract 2x from both sides.

\[
6 \leq 8
\]

Since 6 is always less than or equal to 8, the solution is all real numbers (Fig. 2.13).

Now Try Exercise 37

EXAMPLE 9  Solve the inequality \( 4(x + 1) > x + 5 + 3x \), and graph the solution on a number line.

Solution

\[
4(x + 1) > x + 5 + 3x
\]

Distributive property was used.

\[
4x + 4 > x + 5 + 3x
\]

Like terms were combined.

\[
4x + 4 > 4x + 5
\]

Subtract 4x from both sides.

\[
4 > 5
\]

Since 4 is never greater than 5, the answer is no solution (Fig. 2.14). There is no real number that makes the statement true.

Now Try Exercise 43
EXERCISE SET 2.8  

Concept/Writing Exercises
1. List the four inequality symbols given in this section and write how each is read.

2. Explain the difference between $>$ and $\geq$.

3. Are the following statements true or false? Explain.
   a) $3 > 3$
   b) $3 \geq 3$

4. If $a < b$ is a true statement, must $b > a$ also be a true statement? Explain.

5. When solving an inequality, under what conditions will it be necessary to change the direction of the inequality symbol?

6. List the six rules used to solve inequalities.

7. When solving an inequality, if you obtain the result $3 < 5$, what is the solution?

8. When solving an inequality, if you obtain the result $5 < 2$, what is the solution?

Practice the Skills
9. a) Multiply both sides of $-7 < 3$ by $-4$. $28 > -12$
   b) Divide both sides of $-7 < 3$ by $-4$.

Solve each inequality and graph the solution on a number line.
10. a) Multiply both sides of $12 > -5$ by $-3$.
    b) Divide both sides of $12 > -5$ by $-3$.

   11. $x + 2 > 6$
   12. $x + 9 \geq 6$
   13. $x - 5 > -1$
   14. $x - 3 \geq -9$
   15. $-x + 3 < 8$
   16. $7 < 3 + w$
   17. $8 \leq 2 - r$
   18. $6 \leq -3 - x$
   19. $-2x < 3$
   20. $-12 \geq -3b$
   21. $2x + 3 \leq 5$
   22. $6m - 12 < -12$
   23. $-4x - 3 > 5$
   24. $7x - 4 < 9$
   25. $4 - 6x > -5$
   26. $8 < 4 - 2x$

   27. $15 > -9x + 50$
   28. $3x - 4 < 5$
   29. $7 > 2x + 10$
   30. $-4x < x + 15$
   31. $6s + 2 \leq 6s - 9$
   32. $-2x - 4 \leq -5x + 12$
   33. $x - 4 \leq 3x + 8$
   34. $-4m > 6 > 4m - 20$
   35. $-x + 4 \leq -3x + 6$
   36. $2(x - 3) < 4x + 10$
   37. $6(2m - 4) \geq 2(6m - 12)$
   38. $-2(w + 3) \leq 4w + 5$
   39. $x + 3 < x + 4$
   40. $y + 4 \geq y - 3$
   41. $6(3 - x) < 2x + 12$
   42. $2(3 - x) + 4x < -6$
   43. $4x - 4 < 4(x - 5)$
   44. $-2(-5 - x) > 3(x + 2) + 4 - x$
   45. $5(2x + 3) \geq 6 + (x + 2) - 2x$
   46. $-3(-2x + 12) < 4(-x + 2) - 6$
   47. $1.2x + 3.1 < 3.5x - 3.8$
   48. $-5.3r - 6.7 \geq 2.3 - 6.5r$
   49. $1.2(m - 3) \geq 4.6(2 - m) + 1.7$
   50. $-4.6(4 - x) < 2.4(x - 3) - 0.2$
   51. $\frac{x}{2} \geq \frac{x}{3} + 5$
   52. $\frac{x}{7} - 1 \geq \frac{x}{8}$
   53. $t + \frac{1}{6} > \frac{2}{3}t$
   54. $\frac{3}{5}t - 9 < \frac{3}{8}$
Chapter 2 Solving Linear Equations and Inequalities

55. \( \frac{1}{5}(4 - r) \leq \frac{1}{4} \)

56. \( 5 - \frac{1}{6}x < \frac{2}{3}x \)

57. \( \frac{2}{3}(t + 2) \leq \frac{1}{4}(2t - 6) \)

58. \( \frac{3}{4}(n - 4) \geq \frac{2}{3}(n - 4) \)

Problem Solving

59. Chicago Temperatures The following chart shows the average high and low monthly temperatures in Chicago over a 126-year period. (Source: Richard Koeneman/WGN-TV meteorologist. Notice that the months are not listed in order.

<table>
<thead>
<tr>
<th>Monthly Average Temperatures</th>
<th>O'Hare International Airport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degrees (Fahrenheit)</td>
<td></td>
</tr>
<tr>
<td>60°</td>
<td>61°</td>
</tr>
</tbody>
</table>

Note: Numbers indicate average monthly highs and lows.

a) In what months was the average high temperature >65°F?
b) In what months was the average high temperature \( \leq 59°F \)?
c) In what months was the average low temperature <29°F?
d) In what months was the average low temperature \( \geq 58°F \)?

60. Hours Worked The following graph indicates the average hours worked per person, per year.

<table>
<thead>
<tr>
<th>Global Hours Worked</th>
<th>Hours worked per year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Korea 2390</td>
<td>0 500 1000 1500 2000 2500</td>
</tr>
<tr>
<td>Mexico 1857</td>
<td></td>
</tr>
<tr>
<td>United States 1792</td>
<td></td>
</tr>
<tr>
<td>France 1431</td>
<td></td>
</tr>
<tr>
<td>Netherlands 1254</td>
<td></td>
</tr>
</tbody>
</table>

Source: Rochester Democrat & Chronicle

Problem Solving

61. The inequality symbols discussed so far are \(<, \leq, >, \geq\), and \(\approx\). Can you name an inequality symbol that we have not mentioned in this section?

62. Consider the inequality \(-4x + 3 \leq 1\). Explain what is wrong with the following solution.

\[
-4x + 3 \leq 1 \\
-4x \leq -2 \\
\frac{-4x}{-4} \geq \frac{-2}{-4} \\
x \geq \frac{1}{2}
\]

63. Consider the inequality \(xy > 6\), where \(x\) and \(y\) represent real numbers. Explain why we cannot do the following step:

\[
\frac{xy}{y} > \frac{6}{y} \\
\text{Divide both sides by } y.
\]

Challenge Problems

64. Solve the following inequality.

\[
3(2 - x) - 4(2x - 3) \leq 6 + 2x - 4x
\]

65. Solve the following inequality.

\[
6x - 6 > -4(x + 3) + 5(x + 6) - x
\]

Cumulative Review Exercises

[1.9] 66. Evaluate \(-x^2\) for \(x = 3\).

[2.7] 68. Solve \(4 - 3(2x - 4) = 5 - (x + 3)\).

[2.7] 69. Electric Bill The Milford Electric Company charges $0.174 per kilowatt-hour of electricity. The Vega’s monthly electric bill was $87 for the month of July. How many kilowatt-hours of electricity did the Vega’s use in July?
## Chapter 2 Summary

<table>
<thead>
<tr>
<th>IMPORTANT FACTS AND CONCEPTS</th>
<th>EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Section 2.1</strong></td>
<td></td>
</tr>
<tr>
<td>The terms of an expression are the parts that are added.</td>
<td>2x^2 − 3xy + 5 has 3 terms: 2x^2, −3xy, and 5.</td>
</tr>
<tr>
<td>The numerical part of a term is called its numerical coefficient.</td>
<td>The numerical coefficient of ( \frac{3x + \frac{3}{4}}{7} ) is ( \frac{3}{4} ).</td>
</tr>
<tr>
<td>A constant is a term that is a number without a variable.</td>
<td>In ( 3x^2 + 2x − 7 ), the −7 is a constant.</td>
</tr>
<tr>
<td><strong>Like terms</strong> have the same variables with the same exponents.</td>
<td>7x and x; 6y^2 and 2y^2; 3(x + 4) and −8(x + 4)</td>
</tr>
<tr>
<td><strong>To Combine Like Terms</strong></td>
<td></td>
</tr>
<tr>
<td>1. Determine which terms are like terms.</td>
<td>−3x^2 + 4y − 7x^2 + 6 − y − 9</td>
</tr>
<tr>
<td>2. Add or subtract the coefficients of the like terms.</td>
<td>= −3x^2 − 7x^2 + 4y − y + 6 − 9</td>
</tr>
<tr>
<td>3. Multiply the number found in step 2 by the common variable(s).</td>
<td>= −10x^2 + 3y − 3</td>
</tr>
<tr>
<td><strong>Distributive Property</strong></td>
<td></td>
</tr>
<tr>
<td>For any real numbers ( a, b ), and ( c ),</td>
<td>(-5(3r − 6) = −15r + 30)</td>
</tr>
<tr>
<td>( a(b + c) = ab + ac )</td>
<td></td>
</tr>
<tr>
<td><strong>To Simplify an Expression</strong></td>
<td></td>
</tr>
<tr>
<td>1. Use the distributive property to remove any parentheses.</td>
<td>( 2(3c − 1) − 5(c + 4) − 6 )</td>
</tr>
<tr>
<td>2. Combine like terms.</td>
<td>( = 6c − 2 − 5c − 20 − 6 )</td>
</tr>
<tr>
<td></td>
<td>( = c − 28 )</td>
</tr>
<tr>
<td>When two or more expressions are multiplied, each expression is a factor of the product.</td>
<td>Since ( 7 \cdot 8 = 56 ), the 7 and the 8 are factors of 56.</td>
</tr>
<tr>
<td><strong>Section 2.2</strong></td>
<td></td>
</tr>
<tr>
<td>A linear equation in one variable is an equation that can be written in the form</td>
<td>9x − 2 = 16</td>
</tr>
<tr>
<td>( ax + b = c )</td>
<td></td>
</tr>
<tr>
<td>where ( a, b ), and ( c ) are real numbers and ( a ≠ 0 ).</td>
<td>The solution to ( 2x + 3 = 9 ) is 3.</td>
</tr>
<tr>
<td>The solution to an equation is the number or numbers that when substituted for the variable or variables make the equation a true statement.</td>
<td>To check whether −2 is the solution to ( −7x + 1 = 15 ):</td>
</tr>
<tr>
<td>The solution to an equation may be checked by substituting the value that is believed to be the solution for the variable in the original equation.</td>
<td>(-7x + 1 = 15)</td>
</tr>
<tr>
<td></td>
<td>(-7(-2) + 1 = \frac{15}{2})</td>
</tr>
<tr>
<td></td>
<td>(14 + 1 = \frac{15}{2})</td>
</tr>
<tr>
<td></td>
<td>(15 = 15 \quad True)</td>
</tr>
<tr>
<td>Thus, −2 is the solution.</td>
<td></td>
</tr>
</tbody>
</table>
## IMPORTANT FACTS AND CONCEPTS

### Section 2.2 (continued)

<table>
<thead>
<tr>
<th>IMPORTANT FACTS AND CONCEPTS</th>
<th>EXAMPLES</th>
</tr>
</thead>
</table>
| **Equivalence** | Two or more equations with the same solution are called **equivalent equations**.  
- \[ -4x = 12, \ 2x - 3 = -9, \text{ and } x = -3 \text{ are equivalent equations} \] |
| **Addition Property of Equality** | If \( a = b \), then \( a + c = b + c \) for any real numbers \( a \), \( b \), and \( c \).  
- Solve the equation \( x - 9 = -2 \).  
  \[
  x - 9 = -2 \\
  x - 9 + 9 = -2 + 9 \\
  x = 7
  \] |
| **Section 2.3** | Two numbers are **reciprocals** of each other when their product is 1.  
- 3 and \( \frac{1}{3} \) are reciprocals since \( 3 \cdot \frac{1}{3} = 1 \). |
| **Multiplication Property of Equality** | If \( a = b \), then \( a \cdot c = b \cdot c \) for any real numbers \( a \), \( b \), and \( c \).  
- Solve the equation \( \frac{3}{7}x = 6 \).  
  \[
  \frac{3}{7}x = 6 \\
  \frac{7}{3} \cdot \frac{3}{7}x = \frac{7}{3} \cdot 6 \\
  x = 14
  \] |
| **Section 2.4** | To Solve Linear Equations with a Variable on Only One Side of the Equal Sign  
1. If the equation contains fractions, multiply both sides of the equation by the least common denominator (LCD).  
2. Use the distributive property to remove parentheses.  
3. Combine like terms on the same side of the equal sign.  
4. Use the addition property to obtain an equation with the term containing the variable on one side of the equal sign and a constant on the other side.  
5. Use the multiplication property to isolate the variable.  
6. Check the solution in the original equation.  
- Solve the equation \( 3(x - 5) - 6x = -2 \).  
  \[
  3(x - 5) - 6x = -2 \\
  3x - 15 - 6x = -2 \\
  -3x - 15 = -2 \\
  -3x - 15 + 15 = -2 + 15 \\
  -3x = 13 \\
  \frac{-3x}{-3} = \frac{-2 + 15}{-3} \\
  x = \frac{-13}{3}
  \]  
A check will show that \( \frac{-13}{3} \) is the solution. |
| **Section 2.5** | To Solve Linear Equations with the Variable on Both Sides of the Equal Sign  
1. If the equation contains fractions, multiply both sides of the equation by the LCD.  
2. Use the distributive property to remove parentheses.  
3. Combine like terms on the same side of the equal sign.  
4. Use the addition property to rewrite the equation with all terms containing the variable on one side of the equal sign and all terms not containing the variable on the other side of the equal sign.  
5. Use the multiplication property to isolate the variable.  
6. Check the solution in the original equation.  
- Solve the equation \( 9 - 3x - 2(x + 5) = 4x + 7 - x \).  
  \[
  9 - 3x - 2(x + 5) = 4x + 7 - x \\
  9 - 3x - 2x - 10 = 4x + 7 - x \\
  -5x - 1 = 3x + 7 \\
  -5x + 5x - 1 = 3x + 5x + 7 \\
  -1 = 8x + 7 \\
  -1 - 7 = 8x + 7 - 7 \\
  -8 = 8x \\
  \frac{-8}{8} = \frac{8x}{8} \\
  -1 = x
  \]  
A check will show that \(-1\) is the solution. |
### IMPORTANT FACTS AND CONCEPTS

#### Section 2.5 (continued)

A **conditional equation** is an equation that has a single value for a solution.

An **identity** is an equation that is true for infinitely many values of the variable.

A **contradiction** is an equation that has no solution.

| Example | 
|---------|---|
| $3x - 2 = 8$ is a conditional equation since its solution is $\frac{10}{3}$. | 
| $-4(x + 3) = -5x - 12 + x$ is an identity because the equation is true for any real number. | 
| $-9x + 7 + 6x = -5x + 1 + 2x$ is a contradiction because the equation is never true and has no solution. | 

#### Section 2.6

**Simple Interest Formula**

\[
\text{interest} = \text{principal} \cdot \text{rate} \cdot \text{time} \quad \text{or} \quad i = prt
\]

Determine the interest earned on a $5000 investment at 3% simple interest for 2 years.

\[
i = 5000(0.03)(2)
i = $300
\]

**Distance Formula**

\[
\text{distance} = \text{rate} \cdot \text{time} \quad \text{or} \quad d = rt
\]

Timothy John completed a snowmobile race in 2.4 hours at an average speed of 75 miles per hour. Determine the distance of the race.

\[
d = (75)(2.4)
d = 180 \text{ miles}
\]

**Area** is the total surface within the figure’s boundaries. Areas are measured in square units.

**Perimeter** is the sum of the lengths of the sides of a figure. Perimeter and area formulas can be found in Table 2.1 on page 142.

Determine the perimeter and area of the following trapezoid.

\[
P = 5 + 6 + 10.4 + 18.6
P = 40 \text{ ft}
A = \frac{1}{2} h (b + d)
A = \frac{1}{2} (4)(6 + 18.6)
A = 49.2 \text{ ft}^2
\]

**The circumference** of a circle is the length (or perimeter) of the curve that forms a circle.

Circle formulas can be found in Table 2.2 on page 144.

Determine the area and circumference of the following circle.

\[
text{Area} = \pi r^2
\text{Area} = \pi (4)^2
\text{Area} = \pi (16)
\text{Area} \approx 50.27 \text{ cm}^2
C = 2\pi r
C = 2\pi (4)
C = 8\pi
C \approx 25.13 \text{ cm}
\]
**Volume** may be considered the space occupied by a figure. Volume is measured in cubic units.

Volume formulas can be found in Table 2.3 on page 145.

To *solve for a variable in a formula*, treat each of the quantities, except the one for which you are solving, as if they were constants. Then solve for the desired variable by isolating it on one side of the equation.

A *ratio* is a quotient of two quantities.

A *proportion* is a statement of equality between two ratios. In the proportion \( \frac{a}{b} = \frac{c}{d} \), the \( a \) and \( d \) are called the *extremes* and the \( b \) and \( c \) are called the *means* of the proportion.

**Cross-Multiplication**

If \( \frac{a}{b} = \frac{c}{d} \), then \( ad = bc \).

**Example**

Determine the volume of the following figure.

Volume may be considered the space occupied by a figure. Volume is measured in cubic units.

Volume formulas can be found in Table 2.3 on page 145.

To solve for a variable in a formula, treat each of the quantities, except the one for which you are solving, as if they were constants. Then solve for the desired variable by isolating it on one side of the equation.

A ratio is a quotient of two quantities.

A proportion is a statement of equality between two ratios. In the proportion \( \frac{a}{b} = \frac{c}{d} \), the \( a \) and \( d \) are called the extremes and the \( b \) and \( c \) are called the means of the proportion.

Cross-Multiplication

If \( \frac{a}{b} = \frac{c}{d} \), then \( ad = bc \).
To Solve Problems Using Proportions

1. Understand the problem.
2. Translate the problem into mathematical language.
3. Carry out the mathematical calculations necessary to solve the problem.
4. Check the answer obtained in step 3.
5. Make sure you have answered the question.

Melanie Jo can type 40 words per minute. If she types for 20.5 minutes, how many words will she type?

\[
\frac{40 \text{ words}}{1 \text{ minute}} = \frac{x \text{ words}}{20.5 \text{ minutes}}
\]

\[820 = x\]

Melanie Jo will type 820 words.

See page 155 for more details on proportions.

Similar figures are figures whose corresponding angles are equal and whose corresponding sides are in proportion.

These two figures are similar.

Section 2.8

An inequality is a mathematical statement containing one or more of the following symbols: >, <, ≥, ≤.

Properties Used to Solve Inequalities

For real numbers, \(a, b,\) and \(c\):

1. If \(a > b\), then \(a + c > b + c\).
2. If \(a > b\), then \(a - c > b - c\).
3. If \(a > b\) and \(c > 0\), then \(ac > bc\).
4. If \(a > b\) and \(c > 0\), then \(\frac{a}{c} > \frac{b}{c}\).
5. If \(a > b\) and \(c < 0\), then \(ac < bc\).
6. If \(a > b\) and \(c < 0\), then \(\frac{a}{c} < \frac{b}{c}\).

1. If \(x - 5 \geq 3\), then \(x - 5 + 5 > 3 + 5\).
2. If \(x + 4 \geq -9\), then \(x + 4 - 4 \geq -9 - 4\).
3. If \(\frac{1}{3}x > 2\), then \(\frac{1}{3}x(3) > 2(3)\).
4. If \(6x > 12\), then \(\frac{6x}{6} > \frac{12}{6}\).
5. If \(-\frac{1}{4}x > 8\), then \((-\frac{1}{4}x)(-4) < 8(-4)\).
6. If \(-7x \geq 35\), then \(-\frac{7x}{-7} \leq \frac{35}{-7}\).

Chapter 2 Review Exercises

[2.1] Use the distributive property to simplify.

1. \(3(x + 4)\)
2. \(5(x - 2)\)
3. \(-2(x + 4)\)
4. \(-(x + 2)\)
5. \(-(m + 3)\)
6. \(-4(4 - x)\)
7. \(5(5 - p)\)
8. \(6(4x - 5)\)
9. \(-5(5x - 5)\)
10. \(4(-x + 3)\)
11. \(\frac{1}{2}(2x + 4)\)
12. \(-\frac{1}{2}(3 + 6y)\)
13. \(-(x + 2y - z)\)
14. \(-3(2a - 5b + 7)\)
Chapter 2  Solving Linear Equations and Inequalities

[2.1] Simplify.
15. 7x - 3x
16. 5 - 3y + 3
17. 1 + 3x + 2x
18. -2x + x + 3y
19. 4m + 2n + 4m + 6n
20. 9x + 3y + 2
21. 6x - 2x + 3y + 6
22. x + 8x - 9x + 3
23. -4x² - 8x² + 3
24. -2(3a² - 4) + 6a² - 8
25. 2x + 3(x + 4) - 5
26. -4 + 2(3 - 2b) + b
27. 6 - (-7x + 6) - 7x
28. 2(2x + 5) - 10 - 4
29. -6(4 - 3x) - 18 + 4x
30. 4y - 3(x + y) + 6x²
31. \( \frac{1}{4} d + 2 - \frac{3}{5} d + 5 \)
32. 3 - (x - y) + (x - y)
33. \( \frac{5}{6} x - \frac{1}{3}(2x - 6) \)
34. \( \frac{2}{3} - \frac{1}{4} - \frac{1}{3}(n + 2) \)

[2.2–2.5] Solve.
35. -3x = -3
36. x + 6 = -7
37. x - 4 = 7
38. \( x + 3 = -9 \)
39. 5x + 1 = 12
40. 14 = 3 + 2x
41. 4c + 3 = -21
42. 9 - 2a = 15
43. -x = -12
44. 3(x - 2) = 6
45. -12 = 3(2x - 8)
46. 4(6 + 2x) = 0
47. -6n + 2n + 6 = 0
48. -3 = 3w - (4w + 6)
49. 6 - (2n + 3) - 4n = 6
50. 4x + 6 = 7x + 9 = 18
51. 5 + 3(x - 1) = 3(x + 1) - 1
52. 8.4r - 6.3 = 6.3 + 2.1r
53. 19.6 - 21.3r = 80.1 - 9.2r
54. 0.35(c - 5) = 0.45(c + 4)
55. 0.2(x + 6) = -0.3(2x - 1)
56. -2.3(x - 8) = 3.7(x + 4)
57. \( \frac{p}{3} + 2 = \frac{1}{4} \)
58. \( \frac{d}{6} + \frac{1}{7} = 2 \)
59. \( \frac{3}{5}(r - 6) = 3r \)
60. \( \frac{2}{3}w = \frac{1}{7}(w - 2) \)
61. 8x - 5 = -4x + 19
62. \( -(w + 2) = 2(3w - 6) \)
63. 2x + 6 = 3x + 9 - 3
64. -5a + 3 = 2a + 10
65. 5p - 2 = -2(-3p + 6)
66. 3x - 12x = 24 - 9x
67. 4(2x - 3) + 4 = 8x - 8
68. 4 - c - 2(4 - 3c) = 3(c - 4)
69. \( 2(x + 7) = 6x + 9 - 4x \)
70. -5(3 - 4x) = -6 + 20x - 9
71. 4(x - 3) - (x + 5) = 0
72. \( -2(4 - x) = 6(x + 2) + 3x \)
73. \( \frac{x + 3}{2} = \frac{x}{2} \)
74. \( \frac{x}{6} = \frac{x - 4}{2} \)
75. \( \frac{1}{5}(3x + 4) = \frac{1}{3}(2x - 8) \)
76. \( \frac{2(2t - 4)}{5} = \frac{3t + 6}{4} - \frac{3}{2} \)
77. \( \frac{2}{5}(2 - x) = \frac{1}{6}(-2x + 2) \)
78. \( \frac{x}{4} + \frac{x}{6} = \frac{1}{2}(x + 3) \)

Determine the area or volume of the figure.
79. y = mx + b (slope-intercept form of a line); find m when y = 7, x = 2, and b = 1.
80. A = \( \frac{1}{2}(b + d) \) (area of a trapezoid); find A when h = 12, b = 3, and d = 5.

Solve for the indicated variable.
81. \( P = 2l + 2w \), for l
82. \( y - y_1 = m(x - x_1) \), for m
83. \( -x + 3y = 2 \), for y
84. Spring Break  Yong Wolfer traveled to Florida for spring break at an average speed of 61.7 miles per hour for 5 hours. How far did he travel?
87. **Flower Garden** Chrishawn Miller has a rectangular flower garden that measures 20 feet by 12 feet. What is the area of Chrishawn’s flower garden?

88. **Tuna Fish** Find the volume of a tuna fish can if its diameter is 4 inches and its height is 2 inches.

[2.7] **Determine the following ratios. Write each ratio in lowest terms.**

89. 12 feet to 20 feet  
90. 80 ounces to 12 pounds  
91. 4 minutes : 40 seconds

Solve each proportion.

92. \( \frac{x}{4} = \frac{8}{16} \)  
93. \( \frac{5}{20} = \frac{x}{80} \)  
94. \( \frac{3}{x} = \frac{15}{45} \)  
95. \( \frac{20}{45} = \frac{15}{x} \)

96. \( \frac{6}{5} = \frac{-12}{x} \)  
97. \( \frac{b}{6} = \frac{8}{-3} \)  
98. \( \frac{-7}{9} = \frac{-12}{y} \)  
99. \( \frac{x}{-15} = \frac{30}{-5} \)

The following pairs of figures are similar. For each pair, find the length of the side indicated by \( x \).

100.  

101.  

[2.8] **Solve each inequality, and graph the solution on a number line.**

102. \( 3x + 4 \leq 10 \)  
103. \( -4a - 6 > 4a - 14 \)

104. \( 5 - 3x \leq 2x + 15 \)  
105. \( 2(x + 4) \leq 2x - 5 \)

106. \( 2(x + 3) > 6x - 4x + 4 \)  
107. \( x + 6 > 9x + 30 \)

108. \( x - 8 \leq -3x + 9 \)  
109. \( -(x + 2) < -2(-2x + 5) \)

110. \( \frac{x}{2} \leq \frac{2}{3}(x + 3) \)  
111. \( \frac{3}{10}(x - 2) = \frac{3}{4}(4 + 2) \)

[2.7] **Set up a proportion and solve each problem.**

112. **Boat Trip** A boat travels 40 miles in 1.8 hours. If it travels at the same rate, how long will it take for it to travel 140 miles?

113. **Washing Dishes** If Adam Kloza can wash 12 dishes in 3.5 minutes, how many dishes can he wash in 21 minutes?

114. **Copy Machine** If a copy machine can copy 20 pages per minute, how many pages can be copied in 22 minutes?

115. **Map Scale** If the scale of a map is 1 inch to 60 miles, what distance on the map represents 380 miles?

116. **Model Car** Bryce Winston builds a model car to a scale of 1 inch to 1.5 feet. If the completed model is 10.5 inches, what is the size of the actual car?

117. **Money Exchange** Suppose that one U.S. dollar can be exchanged for 9.165 Mexican pesos, find the value of 1 peso in terms of U.S. dollars.

118. **Ketchup** If a machine can fill and cap 80 bottles of ketchup in 50 seconds, how many bottles of ketchup can it fill and cap in 2 minutes?
Chapter 2 Practice Test

To find out how well you understand the chapter material, take this practice test. The answers, and the section where the material was initially discussed, are given in the back of the book. Each problem is also fully worked out on the Chapter Test Prep Video CD. Review any questions that you answered incorrectly.

Use the distributive property to simplify.
1. \(-3(4 - 2x)\)
2. \(-(x + 3y - 4)\)

Simplify.
3. \(5x - 8x + 4\)
4. \(4 + 2x - 3x + 6\)
5. \(-y - x - 4x - 6\)
6. \(a - 2b + 6a - 6b - 3\)
7. \(2x^2 + 3 + 2(3x - 2)\)

Solve exercises 8–16.
8. \(2.4x - 3.9 = 3.3\)
9. \(
\frac{5}{6}(x - 2) = x - 3
\)
10. \(2x - 3(-2x + 4) = -13 + x\)
11. \(3x - 4 - x = 2(x + 5)\)
12. \(-3(2x + 3) = -2(3x + 1) - 7\)
13. \(ax + by + c = 0,\) for \(x\)
14. \(-6x + 5y = -2,\) for \(y\)
15. \(
\frac{1}{7}(2x - 5) = \frac{3}{8}x - \frac{5}{7}
\)
16. \(
\frac{9}{x} = \frac{3}{-15}
\)

17. What do we call an equation that has
   a) exactly one solution,
   b) no solution,
   c) all real numbers as its solution?

Cumulative Review Test

Take the following test and check your answers with those given in the back of the book. Review any questions that you answered incorrectly. The section where the material was covered is indicated after the answer.

1. Multiply \(\frac{52}{15} \times \frac{10}{13}\)
2. Divide \(\frac{5}{24} \div \frac{2}{9}\)
3. Insert \(<, >,\) or \(=\) in the shaded area to make a true statement: \(-2\) \(\underline{\hspace{1cm}} 1\).
4. Evaluate \(-5 - (-4) + 12 - 8\).
5. Subtract \(-6\) from \(-7\).
6. Evaluate \(20 - 6 + 3 \cdot 2\).
7. Evaluate \(3(6 - (4 - 3^2)) - 30\).
8. Evaluate \(-2x^2 - 6x + 8\) when \(x = -2\).
9. Name the illustrated property.

Simplify.
10. \(8x + 2y + 4x - y\)
11. \(9 - \frac{2}{3}x + 16 + \frac{3}{4}x\)

12. \(7x + 3 = -4\)
13. \(4(x - 2) = 5(x - 1) + 3x + 2\)
14. \(\frac{3}{4}n - \frac{1}{5} = \frac{2}{3}\)
15. \(A = \frac{a + b + c}{3},\) for \(b\)
16. \(\frac{40}{30} = \frac{3}{x}\)

Solve, and graph the solution on a number line.
17. \(x - 3 > 7\)
18. \(2x - 7 \leq 3x + 5\)
19. Trampoline A circular trampoline has a diameter of 22 feet. Determine the area of the trampoline.
20. Earnings If Samuel earns $10.50 after working for 2 hours mowing a lawn, how much does he earn after 8 hours?